MATHEMATICAL INSTITUTE
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Mathematical Methods for Economists
1st Assignment

Exercise 1 (Field)

Let $\mathbb{F}$ be a field of scalars. Prove the following statement: For any $x \in \mathbb{F}$, $0x = 0,\ -x = (-1)x$.

Exercise 2 (Vector Space)

Consider $\mathbb{R}^2$ and the subset $W = \{(x, y) \in \mathbb{R}^2 | y \geq 0\}$ with the same addition and scalar multiplication. Show that W is not a real vector space (or subspace of $\mathbb{R}^2$).

Exercise 3 (Span)

Prove Theorem 2.15 discussed in the lecture.

Exercise 4 (Linear Independence)

Are the following vectors linearly dependent or linearly independent?

a) $v_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} -6 \\ 3 \\ 0 \\ -9 \end{pmatrix}$

b) $v_1 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$

c) $v_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 7 \end{pmatrix}$

Exercise 5 (Subspace and Basis)

Consider $\mathbb{R}^3$ and the following subset:

$$W = \{(x, y, z) \in \mathbb{R}^3 : 2x - 3y + 3z = 0\}$$

a) Show that W is a subspace of $\mathbb{R}^3$.
b) Find a basis.

Exercise 6 (Basis Reduction)

Consider the following tuple of vectors:

$$(v_1, v_2, v_3, v_4, v_5) = \left(\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \right)$$

a) Define the span of $(v_1, ..., v_5)$ and show that $span(v_1, ..., v_5) = \mathbb{R}^3$.
b) Find a basis for $\mathbb{R}^3$ by an appropriate procedure.

Exercise 7 (Linear Transformation I)

Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}.$$

Show that $nullT = 1$ and $rkT = 2$.

Exercise 8 (Linear Transformation II)

Show by induction that

$$T(\lambda_1 v_1 + ... + \lambda_n v_n) = \lambda_1 T v_1 + ... + \lambda_n T v_n.$$

Exercise 9 (Linear Transformation III)

Prove theorem 2.45 discussed in the lecture.

Exercise 10 (Linear System of Equations)

Consider the following linear system of equations:

$$\begin{aligned} 2x - 4y &= 2 \\ 3x + 4y &= 12 \end{aligned}$$

a) Solve this system by substitution and sketch the row picture.
b) Recognize the linear system as a vector equation. Sketch the column picture by taking the solution of a) into account.

Hint: The row picture illustrates two lines meeting at a single point and the column picture combines vectors on the left-hand side to produce the vector on the right-hand side.