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Mathematical Methods for Economists
2nd Assignment

Exercise 11 (Method of Gaussian Elimination)

Let $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. Determine all solutions to $Ax = b$ by the method of Gaussian elimination.

a) $A = \begin{pmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & -3 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 4 & 3 \end{pmatrix}$, $b = \begin{pmatrix} -8 \\ -20 \\ -2 \\ 4 \end{pmatrix}$

b) $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ 2 & -4 & -2 \end{pmatrix}$, $b = \begin{pmatrix} -2 \\ 6 \\ -6 \end{pmatrix}$

c) $A = \begin{pmatrix} 2 & 5 & -3 \\ 4 & -4 & 1 \\ 4 & -2 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 1 \\ \frac{8}{7} \end{pmatrix}$

Exercise 12 (Linear Transformations and Matrices)

Prove the following statement:
If $A \in \mathbb{M}(m \times n, \mathbb{F})$ and $T_A : \mathbb{F}^n \to \mathbb{F}^m$ is defined by $T_A x = Ax$, then T_A is a linear transformation.

Exercise 13 (Inverse Matrix)

Determine the inverse of the following matrices:

a) $A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$ b) $B = \begin{pmatrix} -3 & 6 \\ 4 & 5 \end{pmatrix}$ c) $C = AB$ d) $D = B^T A^T$ e) $E = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

Exercise 14 (Kernel and Rank of Matrices)

Determine the kernel and rank of the following matrices:

a) $A = \begin{pmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$ b) $B = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ c) $C = \begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$

Furthermore, find a basis for the kernel of these matrices.

Exercise 15 (Column Space)

Consider the following matrices and vectors:

a) $A=\begin{pmatrix} 1 & 3 \\ 4 & -6 \end{pmatrix}$, $b=\begin{pmatrix} -2 \\ 10 \end{pmatrix}$ b) $A=\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$, $b=\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

Decide if the vector b is in the column space of matrix A. In that case, write the vector b as a linear combination of the columns of A.

Exercise 16 (System of Linear Equations)

Consider

$$A=\begin{pmatrix} 1 & 0 & 2 & 3 \\ 1 & 3 & 2 & 0 \\ 2 & 0 & 4 & 9 \end{pmatrix},\ b=\begin{pmatrix} 2 \\ 5 \\ 10 \end{pmatrix}.$$

a) Calculate the solutions to $Ax=0$.
b) Calculate a particular solution to Ax=b.
c) What does the set of solutions look like?

Exercise 17 (Determinant)

Determine the determinant of the following matrices:

a) $A=\begin{pmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{pmatrix}$ b) $B=\begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$ c) $C=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$

Exercise 18 (Keynesian National Income Model)

Consider the Keynesian National Income Model:

Endogenous variables:	Y	national income
	C	consumption expenditure
Exogenous variables:	I_0	investment expenditure
	G_0	government expenditure
Parameters:		$a > 0,\ 0 < b < 1$

$$\text{Model:}\quad \begin{aligned} Y &= C + I_0 + G_0 && \text{(equilibrium condition)} \\ C &= a + bY && \text{(consumption function)} \end{aligned}$$

Find the equilibrium values of the endogenous variables by an appropriate approach.

Exercise 19 (Two-Commodity Market Model)

Consider a two-commodity market model with linear demand and supply functions. Then, the model is given by:

Q_{di}	quantity demanded of commodity i, i=1,2
Q_{si}	quantity supplied of commodity i, i=1,2

$$
\begin{aligned}
Q_{d1} &= a_0 + a_1 P_1 + a_2 P_2 \\
Q_{s1} &= b_0 + b_1 P_1 + b_2 P_2 \\
Q_{d2} &= c_0 + c_1 P_1 + c_2 P_2 \\
Q_{s2} &= d_0 + d_1 P_1 + d_2 P_2
\end{aligned}
$$

a) Find an appropriate equilibrium condition.
b) Determine the equilibrium prices and quantities for the following settings:

$$a_0 = 9,\ a_1 = -1,\ a_2 = 2,\ b_0 = -2,\ b_1 = 4,\ b_2 = 0,$$

$$c_0 = 11,\ c_1 = 2,\ c_2 = -2,\ d_0 = -1,\ d_1 = 2,\ d_2 = 0.$$

First, show that an unique equilibrium exist.