

Mathematical Methods for Economists

3rd Assignment

Exercise 20 (IS-LM Model: Closed Economy)

The IS-LM model is a macroeconomic model that graphically represents two intersecting curves, namely the investment/saving (IS) and liquidity preference/money supply (LM) curves; it is given by the following equations:

Endogenous variables: Y national income
 C consumption expenditure
 I investment expenditure
 r rate of interest

Exogenous variables: G_0 government expenditure
 M_0 stock of money

Parameters: a, d, e, b, t, k, l

Good market: $Y = C + I + G_0$
 $C = a + b(1 - t)Y$
 $I = d - er$

Money market: $M_d = M_s$ (equilibrium condition)
 $M_d = kY - lr$ (money demand)
 $M_s = M_0$ (money supply)

- Find an appropriate equation for the money market.
- To find an equilibrium, formulate a system of 4 equations and solve it.
- Reduce the system in b) by substitution and find an equilibrium.
- Compare the equilibrium values resulting from b) and c).
- Analyze graphically the effect of an increase in G_0 .

Exercise 21 (Partial Market Equilibrium - A Nonlinear Model)

Replacing the linear demand function in the isolated market model introduced in Motivation 2.2.14 (see lecture) by a quadratic demand function leads to the following model:

$$\begin{aligned}Q_d &= Q_s \\Q_d &= a - bP^2 \\Q_s &= -c + dP \\a, b, c, d &> 0\end{aligned}$$

Set $a=4$, $b=1$, $c=1$, $d=4$. Does an (unique) equilibrium exist?

Exercise 22 (Eigenvalues and Eigenvectors)

Find the eigenvalues and corresponding eigenvectors of the following matrices:

$$\text{a) } A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix} \quad \text{b) } B = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \quad \text{c) } C = \begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{pmatrix}$$

Exercise 23 (Matrix Norm)

Show that $\|A\|_2 = \sqrt{\text{largest eigenvalue of } A'A}$.

Exercise 24 (Complex Numbers)

Discuss the complex numbers \mathbb{C} .