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Winter 2009/10
February 2

Mathematical Methods for Economists
4th Assignment

Exercise 25 (One-Commodity model I)

Consider the one-commodity model introduced in subsection 2.2 in the lecture:

$$\begin{aligned} Q &= a - bP \quad \text{(demand)} \\ Q &= -c + dP \quad \text{(supply)} \\ & a, b, c, d > 0 \end{aligned}$$

How will the equilibrium value of the endogenous variables change when there is a change in a,b,c or d? Sketch a comparative-static analysis.

Exercise 26 (Jacobian)

Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$f(x, y, z) = \left(xy - z, \frac{x^2 y}{z}, \frac{xz}{y^2} \right),$$

and $x_0 = (2, 1, -1)$. Determine the Jacobian of f at x_0.

Exercise 27 (Implicit Function Theorem I)

Let $F(x_1, x_2, x_3, x_4) = \begin{pmatrix} 2x_1 - 3x_2 + 4x_3 - x_4 \\ 2x_1 - x_2 + 5x_3 - 2x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Note that $F : \mathbb{R}^4 \to \mathbb{R}^2$ is a linear transformation. Consider any vector (x, y) in $\mathbb{R}^4$ with $F(x, y) = 0$ for $x = (x_1, x_2)$ and $y = (x_3, x_4)$. Show that $J_F(y) \neq 0$ for every y. As a consequence, we can find a function $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $y = f(x)$. Determine $J_f(x)$.

Exercise 28 (Implicit Function Theorem II)

Consider the following system of equations:

$$\begin{aligned} x^2 - 2xz + y^2 z^3 - 7 &= 0 \\ 2xy^4 - 3y^2 + xz^2 + 5z + 2 &= 0 \end{aligned}$$

Let $x_0 = 2$ and $y_0 = (1, -1)$. Can we write $(y, z) = f(x)$ in a neighborhood of 2? If so, determine $J_f(x_0)$ by using

$$J_f(x_0) = -(J_F(y_0))^{-1} J_F(x_0)$$

Exercise 29 (Implicit Function Theorem III)

Let

$$F(x_1, x_2, x_3, x_4, x_5) = \begin{pmatrix} x_1^4 - 2x_3x_4 + x_2x_5^3 + 9 \\ x_1x_2x_3x_4x_5 + 4\frac{x_3x_5}{x_2} \\ \frac{2x_2+5x_4}{x_3} + x_1x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Show that (x_3, x_4, x_5) can be written as a function of (x_1, x_2) in a neighborhood of $(1, -2, 3, -1, 2)$ and compute $J_f(1, -2)$.

Exercise 30 (One-Commodity Model II)

Consider the following one-commodity model:

$$\begin{aligned} Q &= D(P, Y) \quad (D_P < 0;\ D_Y > 0) \\ Q &= S(P, T) \quad (S_P > 0;\ S_T < 0) \end{aligned}$$

with the endogenous variables P (price), Q (quantity), and the exogenous variables Y (income), T (tax). By using the IFT make a comparative-static analysis.