MATHEMATICAL INSTITUTE
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Mathematical Methods for Economists
6th Assignment

Exercise 42 (Local Maximum/Minimum Points and Saddle Points I)

Consider the function

$$f(x, y, z) = x^3 + 3xy + 3xz + y^3 + 3yz + z^3.$$

The points $(-2, -2, -2)$ and $(0, 0, 0)$ are stationary points. Classify theses points.

Exercise 43 (Local Maximum/Minimum Points and Saddle Points II)

Consider the function

$$f : \mathbb{R}^2 \to \mathbb{R},\ f(x, y) = (1 + y)^3 x^2 + y^2.$$

Examine the point $(0, 0)$ and show that f has no global minimum.

Exercise 44 (Lagrange Multiplier as Shadow Prices)

According to the lecture show that $\frac{\partial f^*(b)}{\partial b_j} = \lambda_j(b), \quad j = 1, ..., m.$

Exercise 45 (Envelope Result for Lagrange Problems)

Consider the standard utility maximization problem

$$\max U(x)$$

subject to $p \cdot x = m$ with the commodity vector $x = (x_1, ..., x_n)$. The maximum value of U will depend on the price vector p and income m, i.e. $U^* = U^*(p, m)$, which is called the indirect utility function. Find $\frac{\partial U^*}{\partial m}$ and $\frac{\partial U^*}{\partial p_i}$.

Exercise 46 (Inequality Constraints I)

Solve max $x^2 + 2y$ s.t. $x^2 + y^2 \leq 5$, $y \geq 0$.

Exercise 47 (Inequality Constraints II)

Consider a firm which has L units of labor available and produces three goods whose values per unit of output are a,b, and c, respectively. Producing x,y, and z units of the goods requires αx^2, βy^2, and γz^2 units of labor, respectively. Solve the following problem:

$$\max f(x, y, z) = ax + by + cz$$

subject to

$$g(x, y, z) = \alpha x^2 + \beta y^2 + \gamma z^2 \leq L.$$

The coefficients a,b,c,α, β, and γ are all positive constants. Find the value function and verify

$$\frac{\partial f^*(b)}{\partial b_j} = \lambda_j(b), \ \ j = 1, ..., m$$

in this case.