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Mathematical Methods for Economists
8th Assignment

Exercise 52 (Compound Interest)

Denote by $w(t) > 0$ the wealth held in an investment account at time t. Suppose that $r(t)$ is the interest rate, with interest compounded continuously. Find an ordinary differential equation describing the dynamic of $w(t)$ and solve it for an initial value of the account $w(0) = w_0$.

Exercise 53 (Solow Growth Model)

Consider the Solow Growth Model introduced in Motivation 3.1.1. Classify the resulting ordinary differential equation for the capital per capita and solve it.

Exercise 54 (Logistic Growth)

Denote by $P(t)$ a population at time t. Assume that the average birth rate is given by $\beta > 0$. Furthermore, suppose that the average death rate is proportional to the size of the population, that is, δP with $\delta > 0$; because of the effects of crowding and increased competition for the available food. Then, the growth rate of the population is given by

$$\frac{dP(t)}{dt}.$$

Find an equation for the growth rate per individual. Determine its solution and suppose that an initial value is given, $P(0) = P_0$. Sketch the growth.

Exercise 55 (Partial Market Equilibrium)

Consider the isolated market model introduced in Motivation 2.2.14:

$$\begin{aligned} Q_d &= a - bP \\ Q_s &= -c + dP \\ & \quad a, b, c, d > 0 \end{aligned}$$

Remember that the equilibrium price is given by

$$P^* = \frac{a+c}{b+d}$$

Denote by $P(t)$ the price at time t; $P(0)$ is the price today. Under the assumption that $P(0) \neq P^*$, the equilibrium price P^* is attainable (if ever) only after a due process of adjustment. Given sufficient time for the adjustment process to work itself out, does it tend

to bring price to the equilibrium level P^*?
To answer this question proceed as follows:

a) Find an appropriate differential equation for $P(t)$.
b) Solve your proposed differential equation in order to find the time path $P(t)$.
c) Sketch your solution.