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Mathematical Methods for Economists
9th Assignment

Exercise 56

Solve

$$\max \sum_{t=0}^{T-1} -\frac{2}{3}u_t x_t + \ln x_T,\ x_{t+1} = x_t(1+u_t x_t),\ x(0) = x_0 \in \mathbb{R}^+,\ u_t \geq 0$$

Exercise 57

Solve

$$\max \sum_{t=0}^{3} (1 + x_t - u_t^2)$$

$$x_{t+1} = x_t + u_t,\ t = 0,1,2,\ x_0 = 0,\ u_t \in \mathbb{R}$$

by using

a) dynamic programming,

b) the maximum principle.

Exercise 58

Solve the infinite horizon problem

$$\max \sum_{t=0}^{\infty} \alpha^t (x_t u_t)^{1-\gamma},\ x_{t+1} = x_t(1-u_t)b,\ u_t \in (0,1),\ t = 0,1,2,...,$$

$$b, x_0 > 0,\ \alpha, \gamma \in (0,1),\ \alpha b^{1-\gamma} < 1.$$

Exercise 59

Consider the Macroeconomic Control Problem in the introduction of Optimal Control Theory. Solve the problem for 'x_T free'.

Exercise 60

Solve the control problem

$$\max \int_0^1 x(t)dt,\ \dot{x}(t) = x(t) + u(t),\ x(0) = 0,\ x(1) \text{ free},\ u \in [-1,1]$$