SS 2022

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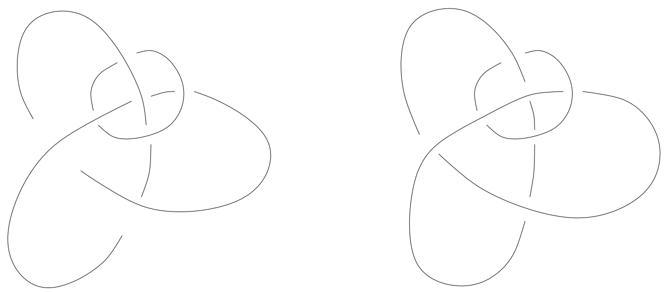
Geometrische Topologie

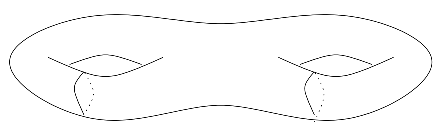
Übungsblatt 6

Aufgabe 1. Show that the following links (in $\mathbb{R}^3$ or S^3) are not isotopic, but that they do have homeomorphic complements.

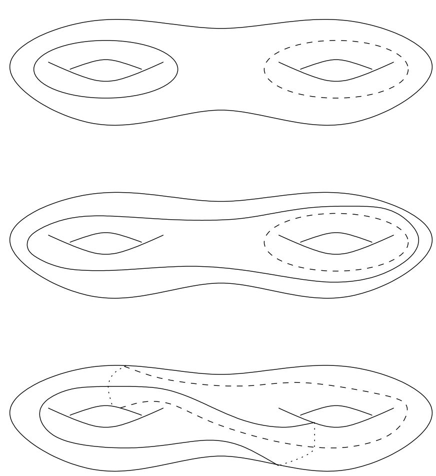


Remark. According to a deep theorem of Gordon and Luecke, *knots* with homeomorphic complements are isotopic.

Aufgabe 2. On the boundary of the handlebody of genus 2, curves u_1, u_2 are chosen as shown.



Prove that the following Heegaard diagrams (each showing $f(u_1), f(u_2)$) all describe the 3-sphere.



Aufgabe 3. The **meridian** μ of the solid torus $S^1 \times D^2$ is the curve $\{*\} \times \partial D^2$, the **longitude** λ, the curve $S^1 \times \{*\}$. Here $*$ denotes an arbitrary point in S^1 or ∂D^2, respectively.

(a) Give an explicit formula (for $k \in \mathbb{Z}$) of a homeomorphism of $S^1 \times D^2$ that sends μ to itself, and λ to a curve of the form $k\mu + \lambda$ (i.e. a curve in the homotopy class $(1, k) \in \mathbb{Z} \oplus \mathbb{Z} = \pi_1(S^1 \times \partial D^2)$).

(b) Show that a homeomorphism of $\partial(S^1 \times D^2)$ extends to a homeomorphism of $S^1 \times D^2$ if and only if μ is sent to $\pm\mu$ (up to isotopy).

(c) Remove a thin open solid torus $S^1 \times \mathrm{Int}(D^2_{1/2})$ from $S^1 \times D^2$.

(i) Show that every homeomorphism f of $\partial(S^1 \times D^2)$ extends to a homeomorphism of $(S^1 \times D^2) \setminus \big(S^1 \times \mathrm{Int}(D^2_{1/2})\big)$.

(ii) Describe a homeomorphism from the solid torus

$$S^1 \times D^2 = \Big((S^1 \times D^2_{1/2}) + (S^1 \times D^2) \setminus \big(S^1 \times \mathrm{Int}(D^2_{1/2})\big)\Big)/x \sim x,$$

where the identification is made for $x \in \partial(S^1 \times D^2_{1/2})$, and the 3-manifold (with boundary)

$$\Big((S^1 \times D^2_{1/2}) + (S^1 \times D^2) \setminus \big(S^1 \times \mathrm{Int}(D^2_{1/2})\big)\Big)/x \sim f(x),$$

where f is regarded as a homeomorphism of $\partial(S^1 \times D^2_{1/2})$.

Why does this not contradict (b)?

Aufgabe 4. Prove the following statements.

(a) Every orientation-preserving homeomorphism of S^1 is isotopic to the identity.

(b) Every homeomorphism of $S^1 \times S^1$ that sends a given meridian μ to itself, and likewise a given longitude λ (not necessarily fixing them pointwise) is isotopic to the identity.

(c) Every Dehn twist of a surface along a simple closed curve that separates the surface (i.e. whose complement has two connected components) is isotopic to the identity.

Abgabe: Mittwoch 18.5.22
bis spätestens 17:00 Uhr im Raum 206/7 des MI