Evarist Giné, David M. Mason and J.A. Wellner (Eds.)
High dimensional probability II

The present volume is an outgrowth of the *Second International Conference on High Dimensional Probability* held at the University of Washington in August 1999. The theme of high dimensional probability serves as a very loose common denominator, which connects a wide range of subjects treated in the 32 research articles in this book. Despite the title, most of the articles of the book are not focussing on more classical subjects of high dimensional probability, like probability in Banach spaces or Gaussian processes, but cover subjects ranging from random walks to testing of symmetry for multivariate distributions, from concentration functions to spline smoothing and from embedding theorems to nonparametric curve estimation. This great variety makes this an interesting book to browse and the articles I have read were well written and of good quality. Although I do not believe that many researchers would consider purchasing the book for their private collection, it is a valuable addition to any mathematical library.

The first chapter of the book is devoted to the general subject of inequalities in probability theory. As an example picked from five contributions to this subject, the article of I. Pinelis discusses the validity of the exact maximal Khinchine equality

$$\mathbb{E} \max_{1\le k\le n} \left| \sum_{i=1}^{k} a_i \varepsilon_i \right|^p \le \mathbb{E} \max_{0\le t\le 1} |B_t|^p$$

relating moments of independent symmetrically distributed signs $\varepsilon_1, \ldots, \varepsilon_n$ and of a standard Brownian motion B for various choices of p and of weights $a_1, \ldots, a_n$ with $\sum a_i^2 = 1$. He shows that the inequality fails to hold in general in a neighbourhood of $p = 1$, disproving a conjecture of Graversen and Peskir, and proves it for $p \ge 3$ if all weights are equal.

One of the topics in this book which deserve special attention are Gaussian processes. In their contribution, W.V. Li and Q.M. Shao discuss the application of the Gaussian correlation conjecture to a small ball problem for fractional Brownian motion. The Gaussian correlation conjecture states that for any two symmetric convex bodies $A, B \subset \mathbb{R}^n$ and any centred Gaussian measure μ on $\mathbb{R}^n$,

$$\mu(A \cap B) \ge \mu(A)\mu(B).$$

The small ball problem is to find the asymptotic rate of decay of $\mathsf{P}\{\sup_{0\le t\le s} |X(t)| \le x\}$ as $x \downarrow 0$ for a centred process X. M.B. Marcus contributes to the less studied lower level problem of finding the rate of decay of $\mathsf{P}\{\sup_{0\le t\le s} X(t) \le x\}$ as $x \downarrow 0$ for a class of Gaussian processes X.

Several contributions of the book deal with problems of embedding and strong approximation. G. Peskir solves the problem of embedding a probability distribution ν in a Brownian motion with drift, generalizing the classical Azéma-Yor embedding. In this very clearly written article he shows that there exists a stopping time τ^* for the Brownian motion with drift $\mu > 0$ such that X_{τ^*} has law ν iff $\int e^{-2\mu x}\nu(dx) \le 1$ and gives an explicit formula for τ^* as well as a minimax characterization. His line of argument should generalize to more general nonrecurrent diffusions.

A substantial amount of the contributions of the book deal with probability problems arising in statistics. In several articles empirical processes and bootstrapping are central themes. For example, A. v.d.Vaart and J.A Wellner present a new result on Glivenko-Cantelli classes of functions. A class $\mathcal{F}$ of functions is called a Glivenko-Cantelli class for a distribution P if the sequence of empirical measures of an iid sequence of random variables with distribution P converge almost surely, uniformly in $\mathcal{F}$ to P. An interesting contribution of C. Haeusler and C. Plies deals with the following problem. Suppose one wants to estimate the distribution function F of an iid sequence $X_1, \ldots, X_n$ of random variables about which one has prior knowledge of the form $\mathbb{E}g(X_1) = 0$ for some $g : \mathbb{R} \to \mathbb{R}^d$. The maximum likelihood estimator for F is then a weighted empirical distribution function. The authors prove a functional central limit theorem in probability for the bootstrap empirical distribution function, which allows to derive asymptotic confidence intervals.

A couple of interesting articles of this book are only very loosely related to high dimensional probability. Two articles on random walks deserve particular attention. Xia Chen studies local times for one-dimensional random walks. He solves a problem of Jain and Pruitt from 1984 by showing that under a very general condition the constants in the laws of the iterated logarithm for the local time in a fixed point and for the supremum over the local times in all points coincide. He also evaluates this constant in the case that the random walk is in the domain of attraction of a not necessarily symmetric stable law.

D. Khoshnevisan and Y. Xiao study images and level sets of additive random walks. An additive random walk is a k-parameter, $\mathbb{Z}^d$-valued process, which can be defined by

$$X(n_1, \ldots, n_k) = X_1(n_1) + \cdots + X_k(n_k),$$

for k independent ordinary random walks $X_1, \ldots, X_k$. An area where additive random walks play a role is the study of intersections of Lévy processes. In the present article, the authors study the size of the random image $X(E)$ of a fixed set E in the domain and apply this to obtain estimates for the probability that the zero set of X intersects E. They also discuss the issue of recurrence and transience of additive random walks.

In order to stress once more the wide range of subjects touched in this book, let me finally mention the interesting contribution of M. Ossiander, which deals with multiplicative random cascades. This is a class of random measures obtained from a tree-indexed family of iid random variables, which have been suggested as models for highly intermittent random phenomena. The present article shows pairwise singularity for a natural parametrised class of random cascade measures.

Altogether, the book contains a good number of interesting and important articles and gives insight into current developments in high dimensional probability and a variety of related subjects. It should not be missing in any good mathematical library.

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