M. Bramson and R. Durrett (Ed.)
Perplexing Problems in Probability
Festschrift in Honor of Harry Kesten

The present *Festschrift* contains a very impressive collection of 21 papers by leading probabilists focussing on current developments in those parts of probability theory, which are most influenced by the work of Harry Kesten. Some of the papers are of expository nature but most of them present new results or new proofs. Throughout the book, the papers satisfy the highest standards in content and presentation, so that the book also serves as a good source of information for the non-specialist in the subjects, who wants to get an overview over current trends and developments in probability. The opening article by R. Durrett reviews the outstanding achievements of Harry Kesten and, by describing his areas of interest in the last 40 years, serves as a guide through the 20 articles that constitute the core of the book.

Much of Kesten's work in the sixties centres around random walks and Lévy processes, turning in the seventies towards subjects like renewal theory and branching processes. These subjects have not lost importance today. This is demonstrated in the book, for example, by a contribution of P.S. Griffin and R.A. Maller who study the connection between weak convergence of random walks and the question when the random walk dominates its largest increment. The contribution of T.M Liggett is devoted to asymptotics of the extinction time of critical and subcritical branching random walks on homogenuous trees.

A particularly interesting contribution is G.F. Lawler's survey on results and conjectures about *loop-erased random walks*. Take a simple random walk on the d-dimensional lattice for $d \geq 2$ and erase loops chronologically from the path. The result, a self-avoiding random path, has quite different features from the usual self-avoiding random walk obtained by taking the uniform distribution on all self-avoiding paths. For example, in $d = 2, 3$, the loop-erased random walk is conjectured to grow faster than the self-avoiding walk. Recently, Wilson gave a simple algorithm to generate uniform spanning trees by running loop-erased random walks on a graph. A highlight of Lawler's paper is a particularly clear derivation of this algorithm.

Starting in the mid-seventies Kesten took great interest in models inspired by physics, like processes in random environments, percolation and first-passage percolation. The subject of no less than eight articles of the present book is directly related to percolation or first-passage percolation.

Percolation means that the edges of an infinite graph are independently removed or retained with probability $1 - p$ or p. The main question is for which values of p the remaining random graph has an infinite connected component with positive probability and whether this component is unique. In the classical situation, when the graph is a Euclidean lattice, it is well known that there is a critical parameter p_c such that for all $p < p_c$ an infinite connected component fails to exist, whereas for $p > p_c$ there is exactly one infinite connected component. Recent research reveals that the situation is different when more general graphs are considered. There exist two critical parameters $p_c \leq p_u$, which may be different, such that for all $p \in (p_c, p_u)$ the number of infinite connected components is infinite and for $p > p_u$ it is unique. The contribution of O. Häggström, Y. Peres and R.H. Schonmann is devoted to the behaviour of the percolation in the regime (p_c, p_u). A recent result of Benjamini and Schramm shows that for a large class of planar graphs at the critical value p_u the infinite cluster is unique.

R.H. SCHONMANN's contribution is the construction of a graph where, at the critical value p_u, there are infinitely many infinite clusters.

The Van den Berg-Kesten (BK) inequality is an important tool in percolation theory. In their article C. BORGS, J.T. CHAYES and D. RANDALL review this inequality, including a proof of the generalization due to Reimer. This is in fact the first published self-contained proof of Reimer's inequality I am aware of.

In *first passage percolation* random passage times are attached to the edges of a graph and important research deals with passage times over large distances, geodesics and the shape of the *wet region*, the set of vertices which can be reached in a given time. The contribution of C.D. HOWARD and C.M. NEWMAN deals with the open problem of existence of geodesics in every asymptotic direction and solves it for the case of Euclidean first-passage percolation. In this model, the vertex set consists of the points of a Poisson process and the passage times between two points are a fixed power of their Euclidean distance.

Beyond models related to percolation, Harry Kesten's influence is particularly strong in the probability theory of stochastic spatial models or interacting particle systems, where he continues to make important contributions. The reader can find a very good cross-section through the current research in this book.

C. NEUHAUSER's beautiful contribution is devoted to stochastic spatial models in plant ecology. Simple non-spatial models of competing species suggest that the number of stably coexisting species is limited by the number of available resources. As plants rely on only a small number of resources, this is in contradiction to the richness of species in plant communities and these theories are unsatisfactory. The author explains how this dilemma can be solved by introducing an implicit or explicit spatial dimension in the models.

Important progress has been made in the last decade in the understanding of the measure-valued process called super-Brownian motion. This process arises as a scaling limit of a system of critical branching random walk particles on the d-dimensional lattice. Much of the recent progress was made possible by the construction of the *Brownian snake* due to J.-F. LE GALL. His contribution presents a typical application of his method to the study of path-properties of super-Brownian motion.

An important recent line of research is the subject of two papers: G. SLADE and T. COX, R. DURRETT and E.A. PERKINS review their newest results about the convergence of rescaled particle systems to super-Brownian motion. These results not only give deep insight into the particle systems themselves, but they also need interesting methods of proof and produce justification for the great amount of recent research about super-Brownian motion.

Of course, I cannot do justice to all the excellent contributions of this volume here. Summarising, the book can be very much recommended to everyone who wants to get an overview over the most topical trends in areas such as percolation theory, stochastic spatial models or interacting particle systems. The excellent quality of the papers of this volume —in presentation as much as in content— are a vivid demonstration of the esteem of Harry Kesten's work and personality in the probability community.

Peter Mörters (Kaiserslautern)