

# International Conference on Modular Forms and $q$ -Series

University of Cologne, March 11-15, 2024

Organizers: Walter Bridges, Kathrin Bringmann, and Johann Franke

All talks are in Lecture Hall (Hörsaal) 2.03 in the math institute, Weyertal 86-90, 50931 Köln

## Monday, March 11

8:15–8:45	<b>Registration</b>		
8:45–8:50	<b>Welcome remarks</b>		
8:50–9:40	Plenary Speaker	<b>Bruce Berndt</b> University of Illinois at Urbana-Champaign	Finite Trigonometric Sums: Evaluations, Estimates, Relations
9:40–9:50	<b>Break</b>		
9:50–10:20	Invited Speaker	<b>Krishnaswami Alladi</b> University of Florida	Alamoudi's bijective proof of Andrews' refinement of the Alladi–Schur Theorem
10:20–10:50	<b>Coffee</b>		
10:50–11:20	Invited Speaker	<b>Ben Kane</b> University of Hong Kong	Explicit Class number formulas for Siegel–Weil averages of ternary quadratic forms
11:20–11:30	<b>Break</b>		
11:30–12:00	Invited Speaker	<b>Armin Straub</b> University of South Alabama	On the representability of sequences as constant terms
12:00–14:00	<b>Lunch</b>		
14:00–14:50	Plenary Speaker	<b>Don Zagier</b> Max Planck Institute for Mathematics, Bonn	Knots, modularity, and $q$ -Series
14:50–15:20	<b>Coffee</b>		
15:20–15:50	Invited Speaker	<b>Ae Ja Yee</b> Pennsylvania State University	The Rogers–Ramanujan Identities and Ariki–Koike Algebras
15:50–16:00	<b>Break</b>		
16:00–16:30	Invited Speaker	<b>Paul Jenkins</b> Brigham Young University	Duality and the trace operator

## Tuesday, March 12

8:50-9:40	Plenary Speaker	<b>Wadim Zudilin</b> Radboud Universiteit Nijmegen	Mahler measures, modular regulators and multiple modular values
9:40-9:50	Break		
9:50-10:20	Invited Speaker	<b>Nick Andersen</b> Brigham Young University	The Weil bound for generalized Kloosterman sums of half-integral weight
10:20-10:50	Coffee		
10:50-11:20	Invited Speaker	<b>Larry Rolen</b> Vanderbilt University	L-functions for Harmonic Maass Forms
11:20-11:30	Break		
11:30-12:00	Invited Speaker	<b>Bernhard Heim</b> University of Cologne	On the Zero Distribution of D'Arcais Polynomials
12:00-14:00	Lunch		
14:00-14:50	Plenary Speaker	<b>Jeremy Lovejoy</b> Université Paris Cité	Bailey pairs and strange identities
14:50-15:20	Coffee		
15:20-15:50	Invited Speaker	<b>Olivia Beckwith</b> Tulane University	Imaginary quadratic fields with $p$ -torsion-free class groups and specified split primes
15:50-16:00	Break		
16:00-17:00	Speed talks		

## Wednesday, March 13

8:50-9:40	Plenary Speaker	<b>Ole Warnaar</b> University of Queensland	The Capparelli-Meurman-Primc-Primc conjectures
9:40-9:50	Break		
9:50-10:20	Invited Speaker	<b>Amanda Folsom</b> Amherst College	Quantum Jacobi forms and $q$ -series
10:20-10:50	Coffee		
10:50-11:20	Invited Speaker	<b>Scott Ahlgren</b> University of Illinois at Urbana-Champaign	Congruences for half-integral weight modular forms
11:20-11:30	Break		
11:30-12:20	Plenary Speaker	<b>Ken Ono</b> University of Virginia	The partition function modulo 2 and 4
12:20-17:00	Free afternoon		

## Thursday, March 14

8:50-9:40	Plenary Speaker	<b>Antun Milas</b> State University of New York at Albany	Multiple q-zeta values and characters of vertex algebras
9:40-9:50	Break		
9:50-10:20	Invited Speaker	<b>Josh Males</b> University of Bristol	Conjectures of Andrews on a Nahm-type sum
10:20-10:50	Coffee		
10:50-11:20	Invited Speaker	<b>Pavel Guerzhoy</b> University of Hawaii	Around and beyond Damerell's theorem (p-adic version; supersingular case)
11:20-11:30	Break		
11:30-12:00	Invited Speaker	<b>Frank Garvan</b> University of Florida	Weight $3/2$ Hecke-Rogers series, holomorphic projection and identities for Zagier's higher order mock theta functions
12:00-14:00	Lunch		
14:00-14:50	Plenary Speaker	<b>Shashank Kanade</b> University of Denver	Perspectives on characters of VOAs
14:50-15:20	Conference gathering with refreshments		
15:20-15:50	Invited Speaker	<b>Zafeirakis Zafeirakopoulos</b> University of Geneva	Using Polyhedral Omega to study some integer partition problems
15:50-16:00	Break		
16:00-17:00	Open problem session		

## Friday, March 15

8:50-9:40	Plenary Speaker	<b>Christian Krattenthaler</b> Universität Wien	Congruence properties of Taylor coefficients of modular forms
9:40-9:50	Break		
9:50-10:20	Invited Speaker	<b>Atul Dixit</b> Indian Institute of Technology Gandhinagar	Mordell-Tornheim zeta functions, Hurwitz lift and functional equations of Herglotz-Zagier type functions
10:20-10:50	Coffee		
10:50-11:20	Invited Speaker	<b>Kağan Kurşungöz</b> Sabancı University	Cylindric partitions into distinct parts
11:20-11:30	Break		
11:30-12:00	Invited Speaker	<b>Ali Uncu</b> University of Bath and Austrian Academy of Sciences	Factorial Basis Method for q-Series Applications
12:00-14:00	Lunch		
14:00-14:50	Plenary Speaker	<b>George Andrews</b> Pennsylvania State University	Parity in MacMahon's Partition Analysis
14:50-15:20	Coffee		

15:20–15:50	Invited Speaker	<b>Nicolas Smoot</b> Universität Wien	Classifying Congruence Families
15:50–16:00	<b>Break</b>		
16:00–16:30	Invited Speaker	<b>Peter Paule</b> RISC/Johannes Kepler University	Ramanujan-type Formulae for 1 over pi: A New Algorithmic Wind?

**Monday, March 11**

## **Finite Trigonometric Sums: Evaluations, Estimates, Relations**

**Bruce Berndt**

University of Illinois at Urbana-Champaign

Several kinds of finite trigonometric sums are evaluated in closed form. For certain sums, reciprocity and/or three sum relations are established. Upper bounds are obtained for sums with multiple sines or cosines, and conjectured lower bounds are made. We evaluate analogues of Ramanujan sums, and for one of these, we obtain a theorem similar to the Franel-Landau criterion for the Riemann Hypothesis. Sun Kim and Alexandru Zaharescu are primary collaborators.

## **Alamoudi's bijective proof of Andrews' refinement of the Alladi-Schur Theorem**

**Krishnaswami Alladi**

University of Florida

Schur's celebrated partition theorem of 1926 asserts the equality  $D(n) = C(n)$ , where  $D(n)$  is the number of partitions of  $n$  into parts that differ by  $\geq 3$  and with no consecutive multiples of 3, and  $C(n)$  is the number of partitions on  $n$  into distinct parts which are not multiples of 3. In 1989, Basil Gordon and I generalized and refined this theorem by the *method of weighted words*. In the early 1990s, I mentioned to George Andrews that it would also be worthwhile to investigate another partition function equal to  $D(n)$  and  $C(n)$ , namely,  $A(n)$  = the number of partitions of  $n$  into odd parts that repeat at most twice. Andrews named the equality  $D(n) = A(n)$  as the Alladi-Schur Theorem when he obtained in 2015, the following deep, elegant and surprising refinement of it: *Let  $D(m, n)$  denote the number of partitions of  $n$  of the type enumerated by  $D(n)$ , with the added condition that the number of parts PLUS the number of even parts is  $m$ . Let  $A(m, n)$  denote the number of partitions of  $n$  into  $m$  odd parts that repeat at most twice. Then  $D(m, n) = A(m, n)$ .* Andrews' proof was by the use of generating function polynomials. In this talk I will sketch an intricate bijective proof of Andrews' refinement of the Alladi-Schur Theorem due my PhD student Yazan Alamoudi.

## **Explicit Class number formulas for Siegel-Weil averages of ternary quadratic forms**

**Ben Kane**

University of Hong Kong

In this talk, we consider representations of integers by positive-definite integral ternary quadratic forms. The generating function for these representation numbers, the so-called theta functions, are modular forms. We investigate the Eisenstein series component of these theta functions, writing the coefficients as explicit Hurwitz class numbers. The talk is based on a collaboration with Kathrin Bringmann and another with Daejun Kim and Srimathi Varadharajan.

## **On the representability of sequences as constant terms**

***Armin Straub***

University of South Alabama

Many sequences in combinatorics and number theory can be represented as constant terms of powers of multivariate Laurent polynomials and, therefore, as diagonals of multivariate rational functions. On the other hand, it is an open question, raised by Don Zagier, to classify those diagonals which are constant terms. We provide such a classification in the case of sequences satisfying linear recurrences with constant coefficients. Various related examples, applications and open problems will be given as time permits. This talk is based on joint work with Alin Bostan and Sergey Yurkevich.

## **Knots, modularity, and q-Series**

***Don Zagier***

Max Planck Institute for Mathematics, Bonn

In this talk I will report on some of the arithmetic aspects of quantum invariants of knots and 3-folds and in particular their modularity properties. This work has been ongoing since 2010, in collaboration with Stavros Garoufalidis and several other authors (Frank Calegari, Rinat Kashaev, Peter Scholze, Campbell Wheeler, and Matthias Storzer). Several papers have already appeared online or in print and various others are currently in progress. Among the topics that will be discussed are the Kashaev invariants of knots, the Bloch group of a number field, units in cyclotomic extensions, Nahm sums, holomorphic quantum modularity, and the Habiro ring and its possible generalizations.

# The Rogers–Ramanujan Identities and Ariki–Koike Algebras

*Ae Ja Yee*

Pennsylvania State University

In 2000, Ariki and Mathas showed that the simple modules of the Ariki–Koike algebras  $\mathcal{H}_{\mathbb{C},v;Q_1,\dots,Q_m}(G(m,1,n))$  (when the parameters are roots of unity and  $v \neq 1$ ) are labeled by the so-called Kleshchev multipartitions. This together with Ariki's categorification theorem enabled Ariki and Mathas to obtain the generating function for the number of Kleshchev multipartitions by making use of the Weyl–Kac character formula. In this talk, I will revisit this generating function for  $v = Q_1 = \dots = Q_a = -1, Q_{a+1} = \dots = Q_m = 1$ . This case is particularly interesting, for the corresponding Kleshchev multipartitions have a very close connection to the Rogers–Ramanujan identities. I will discuss an analytic proof of this generating function.

The second objective of this talk is to count the number of simple modules of the Ariki–Koike algebra in a fixed block. It is known that the simple modules of the Ariki–Koike algebras in a fixed block are labeled by the Kleshchev multipartitions with a fixed partition residue statistic. This partition statistic was also studied in the works of Berkovich, Garvan, and Uncu. Employing their results, we can get two bivariate generating function identities for  $m = 2$ . I will also discuss these identities.

This talk is based on joint work with S. Chern, Z. Li, D. Stanton, and T. Xue.

## Duality and the trace operator

*Paul Jenkins*

Brigham Young University

A modular grid is a pair of sequences  $(f_m)$  and  $(g_n)$  of weakly holomorphic modular forms with the property that for almost all  $m$  and  $n$ , the coefficient of  $q^n$  in  $f_m$  is the negative of the coefficient of  $q^m$  in  $g_n$ . This coefficient duality holds for canonical row-reduced bases for a wide variety of spaces of weakly holomorphic modular forms of integral or half-integral weight. New modular grids can also be constructed through various transformations and linear operations on existing modular grids. For integer weight spaces of genus zero, we investigate the effect of the trace operator on modular grids and show exactly when duality still holds after applying the trace operator. This is joint work with Archer Clayton.

**Tuesday, March 12**

## **Mahler measures, modular regulators and multiple modular values**

**Wadim Zudilin**

Radboud Universiteit Nijmegen

Originally designed for needs in the theory of transcendental numbers, the Mahler measure of multivariate polynomials is intrinsically linked with regulators of classes on number fields and curves. In turn, the regulators are related to the  $L$ -values associated with the underlying fields or curves, via Beilinson's conjectures. In our recent work with François Brunault we explicitly computed the Goncharov regulator integral for  $K_4$  classes on modular curves in terms of multiple modular values and reduced the latter to single  $L$ -values of modular forms. As an application of our formula, Brunault resolved a longstanding conjecture of Boyd and Rodriguez Villegas about the Mahler measure of polynomial  $(1+x)(1+y)+z$ . In my talk I will outline principal steps of the machinery in our work with Brunault and its connection with the computation of Mahler measures.

## **The Weil bound for generalized Kloosterman sums of half-integral weight**

**Nick Andersen**

Brigham Young University

Let  $L$  be an even lattice of odd rank with discriminant group  $L'/L$ , and let  $\alpha, \beta \in L'/L$ . We prove the Weil bound for the Kloosterman sums  $S_{\alpha, \beta}(m, n, c)$  of half-integral weight for the Weil Representation attached to  $L$ . We obtain this bound by proving an identity that relates a divisor sum of Kloosterman sums to a sparse exponential sum. This identity generalizes Kohnen's identity for plus space Kloosterman sums with the theta multiplier system.

## **L-functions for Harmonic Maass Forms**

**Larry Rolen**

Vanderbilt University

The theory of harmonic Maass forms and mock modular forms has seen an explosion of activity in the past 20 years, with applications to physics, partitions, enumerative geometry, and many other topics. Along the way, much has been developed in the theory of harmonic Maass forms. However, until recently, harmonic Maass form theory lacked analogues of key structures that exist for classical holomorphic modular forms and Maass waveforms, such as the theory of L-functions. Recent work with Diamantis, Lee and Raji will be described which gives the first general such theory. In particular, I will sketch Weil-type Converse Theorems and a Voronoi-type summation formula in these settings. I will also describe connections with the construction of differential operators on these spaces and a more thorough explanation of a previous formula for a central L-value of the  $\mu$ -invariant, which had been discovered heuristically by Zagier and proven in that case by Bruinier, Funke, and Imamoglu.



## **On the Zero Distribution of D'Arcais Polynomials**

***Bernhard Heim***

Universität zu Köln

The D'Arcais polynomials (shifted Nekrasov-Okounkov polynomials) dictate the vanishing properties of the  $q$ -expansion of powers of the Dedekind eta function. These polynomials have remarkable properties. They can be defined also via a hereditary difference equation, generalizing Euler's well-known recursion for the partition numbers. This leads to several interesting applications and generalizations. This includes non-vanishing results for the powers of the Dedekind eta function, results towards orthogonal polynomials and reciprocals of Eisenstein series as considered by Hardy and Ramanujan. This is joint work with Markus Neuhauser.

## **Bailey pairs and strange identities**

***Jeremy Lovejoy***

Université Paris Cité

Zagier introduced the term "strange identity" to describe an asymptotic relation between a certain  $q$ -hypergeometric series and a partial theta function at roots of unity. We show that behind Zagier's strange identity lies a statement about Bailey pairs. Using the iterative machinery of Bailey pairs then leads to many families of multisum strange identities, including Hikami's generalization of Zagier's strange identity. As an application, we obtain some families of  $q$ -series identities at roots of unity.

## **Imaginary quadratic fields with $p$ -torsion-free class groups and specified split primes**

***Olivia Beckwith***

Tulane University

We study Ramanujan-type congruences for Hurwitz class numbers. As an application, we show that for any odd prime  $p$  and finite set of odd primes  $S$ , there exists an imaginary quadratic field which splits at each prime in  $S$  and has class number indivisible by  $p$ . This result is in the spirit of results by Bruinier, Bhargava (when  $p = 3$ ) and Wiles, but the methods are completely different. This is joint work with Martin Raum and Olav Richter.

# Wednesday, March 13

## The Capparelli–Meurman–Primc–Primc conjectures

*Ole Warnaar*

University of Queensland

In 2022/23 Capparelli, Meurman, Primc and Primc conjectured three beautiful families of partition identities, one of which generalises Gordon's celebrated theorem to partitions in which the parts can take  $n$  distinct colours. In this talk I will describe all three conjectures, explain how they are related to the affine Lie algebras  $A_{2n}^{(2)}$ ,  $C_n^{(1)}$  and  $D_{n+1}^{(2)}$ , and connect special cases of each of the conjectures to the generalised Rogers–Ramanujan identities of Griffin, Ono and the speaker, thereby providing a simple combinatorial interpretation of the GOW identities. This talk is based on joint work with Shashank Kanade, Matthew Russell and Shunsuke Tsuchioka.

## Quantum Jacobi forms and $q$ -series

*Amanda Folsom*

Amherst College

We establish quantum Jacobi properties of diverse families of  $q$ -(hypergeometric) series appearing in work of Hikami, Kirillov, Lovejoy, Milas et al, Osburn et al, and others, some of which arise from knot invariants, vertex algebra characters, and so-called "strange" identities. We show how these quantum Jacobi families are unified by a general infinite family of two-variable periodic partial theta functions, whose modular properties we explicitly establish.

## Congruences for half-integral weight modular forms

*Scott Ahlgren*

University of Illinois at Urbana-Champaign

I will survey some recent results with several co-authors (Olivia Beckwith, Martin Raum, Patrick Allen, Shiang Tang, Nick Andersen and Robert Dicks) which provide new understanding about congruences for modular forms of half-integral weight which transform like a power of the Dedekind eta function. As an application these results shed light on the problem of understanding linear congruences for the ordinary partition function.

## The partition function modulo 2 and 4

*Ken Ono*

The University of Virginia

The Ramanujan congruences for the partition function have an extraordinary legacy in mathematics. These days research abounds with new congruences for various restricted partition functions. Unfortunately very little is known about  $p(n)$  modulo powers of 2. In this talk the speaker will discuss new and old results about the partition function modulo 2 and 4, and will offer a few precise open questions with the idea of catalyzing work in the area.

## Thursday, March 14

### Multiple $q$ -zeta values and characters of vertex algebras

*Antun Milas*

State University of New York at Albany

We discuss multiple  $q$ -zeta values and their generalizations within the contexts of algebraic geometry, specifically arc schemes, and in the framework of vertex algebras. Notably, we introduce multiple  $q$ -zeta values associated with simple Lie algebras and discuss their conjectured properties.

### Conjectures of Andrews on a Nahm-type sum

*Joshua Males*

University of Bristol

In the 1980s Andrews introduced and made conjectures on several  $q$ -series arising from combinatorics. The most famous of these is the  $\sigma$ -function, whose coefficients are now known to be intricately linked to the arithmetic of the field  $\mathbb{Q}(\sqrt{6})$  by Andrews-Dyson-Hickerson, and is also a first example of Zweger's mock Maass theta functions and Zagier's quantum modular forms.

In the same paper that  $\sigma$  was introduced, Andrews made conjectures on several other Nahm-type sums which were slightly left behind in the shadow of their sibling  $\sigma$ . In joint work with Folsom, Rolin, and Storzer, we prove two conjectures of Andrews on one of these Nahm-type sums, and offer explanations of two further conjectures, ultimately obtaining glimpses of its relationship to the field  $\mathbb{Q}(\sqrt{-3})$ .

### Around and beyond Damerell's theorem (p-adic version; supersingular case)

*Pavel Guerzhoy*

University of Hawaii

We discuss a definition for  $p$ -adic analogs for the values of the weight 2 Eisenstein series at lattices corresponding to elliptic curves with both ordinary and supersingular reduction at  $p$ .

We also discuss another (somewhat similar in appearance but actually quite different)  $p$ -adic quantity and its provable non-vanishing. As an illustration, in the special case of the CM elliptic curve of conductor 32, this quantity becomes the limit of simple ratios of Catalan numbers, and can be evaluated in a closed form in terms of Morita's  $p$ -adic Gamma-function.

# Weight 3/2 Hecke-Rogers series, holomorphic projection and identities for Zagier's higher order mock theta functions

*Frank Garvan*

University of Florida

Andrews discovered Hecke-Rogers identities for Ramanujan's fifth and seventh order mock theta functions. This enabled Zwegers to complete them as holomorphic parts of real analytic vector modular forms on  $SL_2(\mathbb{Z})$ . In a talk in May, 2009, Zagier indicated how these results could be generalized using holomorphic projection. We give details of these generalizations. This leads to new identities for Ramanujan's mock theta functions in terms of weight 3/2 Hecke-Rogers series and connects Zagier's 11th order mock theta functions with Dyson rank differences mod 11 in the same way that the Mock Theta Conjectures of Andrews and the speaker connect Ramanujan's mock theta functions of order 5 with Dyson rank differences mod 5.

## Perspectives on characters of VOAs

*Shashank Kanade*

University of Denver

In this talk, I will explore knot-theoretic and combinatorial perspectives on the characters of various VOAs and their modules.

I will discuss how characters of rational and non-rational VOAs relate to the coloured Jones invariants of torus links via a new combinatorial conjecture on asymptotic weight multiplicities. The ultimate goal is to gain a knot-theoretic understanding of Rogers–Ramanujan-type identities for VOA characters, especially beyond the  $\mathfrak{sl}_2$  cases.

On the combinatorial side, Warnaar has recently made significant progress on the  $\mathfrak{sl}_3$  Bailey machinery, while proving our conjectures with Russell regarding the Rogers–Ramanujan identities for the  $\mathcal{W}_3$  VOA. Progress is also underway for new character identities for certain non-rational VOAs based on  $\mathfrak{sl}_3$ . I will provide an overview of these developments.

# Using Polyhedral Omega to study some integer partition problems

*Zafeirakis Zafeirakopoulos*

University of Geneva

MacMahon developed Partition Analysis in order to compute generating functions for integer partitions under linear constraints, i.e., a multivariate rational function representation of the set of all non-negative integer solutions to a system of linear equations and inequalities. Polyhedral Omega combines methods from partition analysis with methods from polyhedral geometry. In particular, it combines MacMahon's iterative approach based on the Omega operator and explicit formulas for its evaluation with geometric tools such as Brion decompositions and Barvinok's short rational function representations. After presenting Polyhedral Omega shortly, we will use it to have a geometric view on certain integer partition problems.

**Friday, March 15**

## **Congruence properties of Taylor coefficients of modular forms**

**Christian Krattenthaler**

Universität Wien

About five years ago, Dan Romik considered the Taylor coefficients of Jacobi's theta function  $\theta_3(\tau)$  at  $\tau = i$  and showed that, under a certain normalisation, these coefficients become integers. He conjectured that, when taken modulo any fixed prime power, these coefficients are eventually periodic, and if  $p$  is congruent to 3 modulo 4 then, more precisely, these coefficients eventually vanish modulo any fixed power of  $p$ .

I shall first explain that, in a sense, periodicity was already known (in a more general context), but very hidden (and nobody noticed). The corresponding argument however only provides astronomic bounds on the period length, and it can not address whether the sequence eventually vanishes modulo a prime power.

I shall then present joint work with Thomas W. Müller in which we prove all of Romik's (not-always-)conjectures together with reasonably good bounds on period lengths respectively on start of vanishing. Furthermore, we show that similar results hold for Jacobi's theta function  $\theta_2(\tau)$ , all Eisenstein series and even weight modular forms.

I shall close with some conjectures on "actual" period lengths respectively on "actual" start of vanishing.

## **Mordell-Tornheim zeta functions, Hurwitz lift and functional equations of Herglotz-Zagier type functions**

**Atul Dixit**

Indian Institute of Technology Gandhinagar

In this talk, we will present our recent results on a generalization of the Mordell-Tornheim zeta function, in particular, the two- and three-term functional equations that it satisfies. This function is intimately connected with a new extension of the Herglotz-Zagier function  $F(x)$ . Radchenko and Zagier recently studied arithmetic properties of  $F(x)$ , in particular, their special values and functional equations coming from Hecke operators. One of our results on this extension not only gives the well-known two-term functional equation of  $F(x)$  as a special case but also those of Ishibashi functions, which were unknown for about twenty years. Some interesting results on integrals involving logarithm and/or arctangent, and related to the generalized Mordell-Tornheim zeta function will also be given. This is joint work with Sumukha Sathyanarayana and N. Guru Sharan.

## **Cylindric partitions into distinct parts**

***Kağan Kurşungöz***

Sabancı University

We will describe an alternative way to describe cylindric partitions. Then, we will present an existence result for generating functions of cylindric partitions, and give some examples. This is joint work with Halime Ömrüzün Seyrek. In particular, the fourth chapter of the preprint at <https://arxiv.org/pdf/2308.14514>.

## **Factorial Basis Method for q-Series Applications**

***Ali Uncu***

University of Bath and Austrian Academy of Sciences

Many combinatorics questions trickle down to finding a formula for a particular solution of some recurrence relation. This is often called the inverse Zeilberger problem. The Factorial Basis method provides solutions to linear recurrence equations in the form of definite sums. We will demonstrate the  $q$ -analog of this method and apply this extended technique to automatically prove identities and unveil novel ones, particularly some associated with the Rogers-Ramanujan identities.

This is joint work with Antonio Jimenez-Pastor.

## **Parity in MacMahon's Partition Analysis**

***George Andrews***

Pennsylvania State University

This is joint work with Peter Paule. The object is to illustrate how parity may be used in applications of P.A. MacMahon's Partition Analysis. We begin with a description of how Partition Analysis with a parity component leads us directly to a combinatorial understanding of the  $q$ -series in the first Goellnitz-Gordon identity. We also show how the addition of parity to Partition Analysis leads to a new interpretation of Hei-Chi Chan's "cubic partitions," and we look at Schmidt-type theorems with applications to overpartitions. We also consider some of the more esoteric theorems arising from this new aspect of Partition Analysis. We close with a look at the arithmetic properties of some of the partition functions that have arisen.



## **Classifying Congruence Families**

***Nicolas Smoot***

Universität Wien

Since Ramanujan's groundbreaking work, the occurrence of congruence families modulo arbitrarily large powers of primes has been an extremely fertile subject area. Many of these congruence families can be proved with routine methods, while others are much more difficult to resolve. A very frustrating issue with our current methods is that it is not always known whether a proof can actually be completed until the very late stages of a long investigation. We give a conjectured classification of congruence families in which the topology of the associated modular curve can tell us whether a family is accessible to our known proof methods, or if it is likely to be more difficult. We show how this classification can be useful in proving congruence problems by examining some recent results.

## **Ramanujan-type Formulae for $1/\pi$ : A New Algorithmic Wind?**

***Peter Paule***

RISC/Johannes Kepler University

In 1914 Ramanujan recorded a list of 17 series for  $1/\pi$ . In his 2008 article "Ramanujan-type formulae for  $1/\pi$ : A second wind?" Wadim Zudilin surveyed methods of proof of Ramanujan's formulae and discussed various generalizations and new discoveries. Since then Zudilin's prediction of a "second wind" has been proven by many further developments. In this talk, which arose from joint work with Ralf Hemmecke (RISC) and Cristian-Silviu Radu (RISC), we present new computer algebra tools in the hope to contribute to a "third wind" of research in this area.