t-structures, recollements and silting objects

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Overview

1. t-structures and co-t-structures;
2. Recollements and glueing;
3. The piecewise hereditary case;
4. Silting objects;
5. Glueing of silting objects;
6. Glueing of tilting objects.

t-structures and co-t-structures

$\mathcal{D}$ triangulated.

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$$A \longrightarrow X \longrightarrow B \longrightarrow A[1]$$

with $A \in \mathcal{D}^{\leq 0}$ and $B \in \mathcal{D}^{\geq 0}[-1]$.

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Definition (Bondarko, Pauksztello'10)

A **co-t-structure** in $\mathcal{D}$ is a pair of full subcategories $(\mathcal{D}_{\geq 0}, \mathcal{D}_{\leq 0})$ s.t.

1. $\mathrm{Hom}_{\mathcal{D}}(\mathcal{D}_{\geq 0}, \mathcal{D}_{\leq 0}[1]) = 0$;
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$$(\mathcal{R}): \quad \begin{tikzcd} \mathcal{Y} \arrow[r, "i_*"] & \mathcal{D} \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^*"] & \mathcal{X} \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd} .$$

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satisfying:

1. $(i^*, i_*, i^!)$, $(j_!, j^*, j_*)$ are adjoint triples;
2. i_*, j_*, $j_!$ are fully faithful;
3. $j^* i_* = 0$;
4. $\forall X \in \mathcal{D}$, there are triangles given by the (co)units of the adjunctions:

$$
i_* i^! X \to X \to j_* j^* X \to i_* i^! X[1] \quad , \quad j_! j^* X \to X \to i_* i^* X \to j_! j^* X[1].
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where B is also a f.d. piecewise hereditary and $End_{\mathcal{D}^b(A)}(X)$ is a f.d. skew-field over $\mathbb{K}$.

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Theorem (Beilinson-Bernstein-Deligne'82)

Given $\mathcal{R}$ and $(\mathcal{X}^{\leq 0}, \mathcal{X}^{\geq 0})$, $(\mathcal{Y}^{\leq 0}, \mathcal{Y}^{\geq 0})$ t-structures in $\mathcal{X}$ and $\mathcal{Y}$, then

$$\mathcal{D}^{\leq 0} := \left\{ Z \in \mathcal{D} : j^* Z \in \mathcal{X}^{\leq 0},\ i^* Z \in \mathcal{Y}^{\leq 0} \right\}$$

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Lemma

Let $(\mathcal{D}^{\leq 0}, \mathcal{D}^{\geq 0})$ be a bounded t-structure with length heart in $\mathcal{D}^b(R)$. Then $\mathcal{D}^{\leq 0} \cap \mathcal{D}^{\geq 0} \cong mod(S)$, where S is a f.d. directed algebra over $\mathbb{K}$.

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Theorem (Liu-V.'11)

Let $(\mathcal{D}^{\leq 0}, \mathcal{D}^{\geq 0})$ be a bounded t-structure with length heart in $\mathcal{D}^b(R)$. Then there is a recollement of $\mathcal{D}^b(R)$ by derived module categories such that $(\mathcal{D}^{\leq 0}, \mathcal{D}^{\geq 0})$ is glued with respect to this recollement.

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for some piecewise hereditary algebra B;

- *$(\mathcal{D}^{\leq 0}, \mathcal{D}^{\geq 0})$ is glued with respect to this recollement from t-structures $(j^*\mathcal{D}^{\leq 0}, j^*\mathcal{D}^{\geq 0})$ and $(i^*\mathcal{D}^{\leq 0}, i^!\mathcal{D}^{\geq 0})$*

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Theorem (Keller-Vossieck'88)

Q Dynkin quiver. The bounded t-structures in $\mathcal{D}^b(\mathbb{K}Q)$ are in bijection with basic silting objects in $\mathcal{D}^b(\mathbb{K}Q)$.

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Assume $gldim(R) < \infty$ $(\Rightarrow \mathcal{K}^b(proj(R)) \cong \mathcal{D}^b(R))$.

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- co-t-structure: $(\mathcal{D}_{\geq 0}, \mathcal{D}_{\leq 0}) \rightsquigarrow$ silting: M s.t. $add(M) = \mathcal{D}_{\geq 0} \cap \mathcal{D}_{\leq 0}$.
- t-structure: $(\mathcal{D}^{\leq 0}, \mathcal{D}^{\geq 0}) \rightsquigarrow$ co-t-structure: $(({}^{\perp}\mathcal{D}^{\leq 0})[1], \mathcal{D}^{\leq 0})$.
- silting: $M \rightsquigarrow$ t-structure:
  $\mathcal{D}_M^{\leq 0} = \{X \in \mathcal{D}^b(R) : Hom(M, X[i]) = 0, \forall i > 0\}$
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Silting objects

Example (t-structure in $\mathcal{D}^b(\mathbb{K}A_3)$, $A_3 = \; 1 \longrightarrow 2 \longrightarrow 3$)

$$\begin{tikzcd}
S_2[-1] \arrow[dr] & & S_3[-1] \arrow[dr] & & P_3 \arrow[dr] & & P_1[1] \arrow[dr] & & S_2[1] \arrow[dr] & & S_3[1] \\
\cdots & I_2[-1] \arrow[ur] \arrow[dr] & & P_2 \arrow[ur] \arrow[dr] & & I_2 \arrow[ur] \arrow[dr] & & P_2[1] \arrow[ur] \arrow[dr] & & I_2[1] \arrow[ur] \arrow[dr] & \cdots \\
P_3[-1] \arrow[ur] & & P_1 \arrow[ur] & & S_2 \arrow[ur] & & S_3 \arrow[ur] & & P_3[1] \arrow[ur] & & P_1[2]
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In general:

- The heart of $(\mathcal{D}_M^{\leq 0}, \mathcal{D}_M^{\geq 0})$ is $\mathrm{End}_{\mathcal{D}^b(R)}(M)$;
- M is tilting if and only if $M \in \mathcal{D}_M^{\leq 0} \cap \mathcal{D}_M^{\geq 0}$.

Silting objects

Example (co-t-structure in $\mathcal{D}^b(\mathbb{K}A_3)$, $A_3 = \; 1 \longrightarrow 2 \longrightarrow 3$)

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Glueing silting

Theorem (Liu-V.-Yang'12)

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- *$\mathcal{R}$ recollement of triangulated category $\mathcal{D}$;*

$$(\mathcal{R}): \quad \begin{tikzcd} \mathcal{Y} \arrow[r, "i_*"] & \mathcal{D} \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^*"] & \mathcal{X} \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd};$$

- *$X \in \mathcal{X}$ and $Y \in \mathcal{Y}$ silting objects;*
- *$(\mathcal{X}_{\geq 0}, \mathcal{X}_{\leq 0})$ and $(\mathcal{Y}_{\geq 0}, \mathcal{Y}_{\leq 0})$ corresponding co-t-structures in $\mathcal{X}$ and $\mathcal{Y}$.*

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- $(\mathcal{X}_{\geq 0}, \mathcal{X}_{\leq 0})$ *and* $(\mathcal{Y}_{\geq 0}, \mathcal{Y}_{\leq 0})$ *corresponding co-t-structures in* $\mathcal{X}$ *and* $\mathcal{Y}$*.*

Then the glued co-t-structure $(\mathcal{D}_{\geq 0}, \mathcal{D}_{\leq 0})$ *corresponds to the silting* $Z = i_*Y \oplus K_X$*, where* K_X *is defined by*

$$i_*\beta_{\geq 1} i^! j_! X \longrightarrow j_! X \longrightarrow K_X \longrightarrow (i_*\beta_{\geq 1} i^! j_! X)[1]$$

($\beta_{\geq 1}$ *is a (non-functorial) choice of truncation for* $(\mathcal{Y}_{\geq 0}, \mathcal{Y}_{\leq 0})$ *in* $\mathcal{Y}$*).*

Glueing silting

Idea of proof

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- *[BBD'82]* $\rightsquigarrow$ *functor* $j_{!*}$ *(7th/intermediate functor)* $\rightsquigarrow$ *glueing simples in the hearts of t-structures;*
- $\{$*Summands of glued silting*$\}$ $\leftrightarrow$ $\{$*"simples" of glued coheart*$\}$*;*
- *"Triangulated" description of 7th functor (using truncations);*
- *Apply it to "simples" of the coheart of* $\mathcal{X}$*;*
- *It works as described!*

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Remark

Glueing silting objects <u>means</u> glueing compatibly with the glueing of co-t-structures.

Glueing silting

Remark

$\mathcal{R}$ recollement of $\mathcal{D}^b(R)$, R of finite global dimension;

- *There is Serre functor in $\mathcal{D}^b(R)$;*
- *Recollements can be reflected (Jørgensen);*

$\rightsquigarrow$ *Glueing of silting can be done, in this setting, compatibly with the glueing of t-structures.*

This corresponds to glueing with respect to certain co-t-structures via a reflected recollement.

Glueing tilting

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Recall that, in $\mathcal{D}^b(R)$ a silting is tilting if and only if it lies in the heart of the corresponding t-structure.

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Theorem (Liu-V.-Yang'12)

- *$\mathcal{R}$ recollement of $\mathcal{D}^b(R)$ by $\mathcal{D}^b(C)$ and $\mathcal{D}^b(B)$*

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Then $Z = i_* Y \oplus K_X$ is tilting if and only if

(a) *$Hom_{\mathcal{D}^b(B)}(Y, i^* j_* X[k]) = 0$ for all $k < -1$;*

(b) *$Hom_{\mathcal{D}^b(B)}(i^* j_* X, Y[k]) = 0$ for all $k < 0$;*

(c) *$Hom_{\mathcal{D}^b(B)}(i^* j_* X, i^* j_* X[k]) = 0$ for all $k < -1$.*

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- *B hereditary.*

Then $Z = i_*Y \oplus K_X$ is tilting if and only if $\exists\ X'_{-1}, X'_0, X'_1, X'_2 \in mod(B)$:

- *X'_2 projective;*
- *$i^*j_*X \cong X'_{-1}[1] \oplus X'_0 \oplus X'_1[-1] \oplus X'_2[-2]$;*
- *$Hom_B(X_m, X_n) = 0$ whenever $n - m \geq 2$.*

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Thank you for your attention.