The Roquette category of finite p-groups

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Such a subgroup S is called a genetic subgroup of G.

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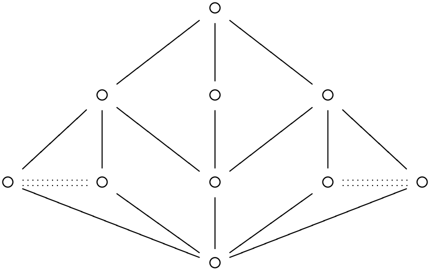

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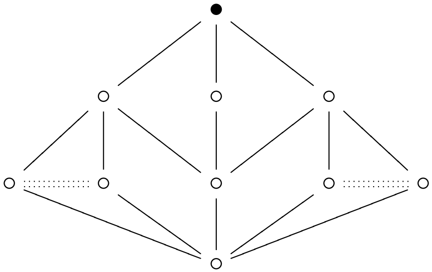

D_8

An example

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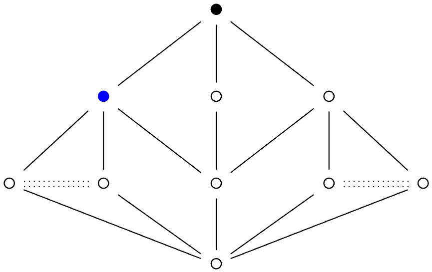

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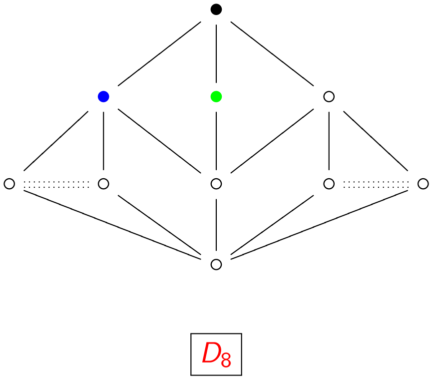

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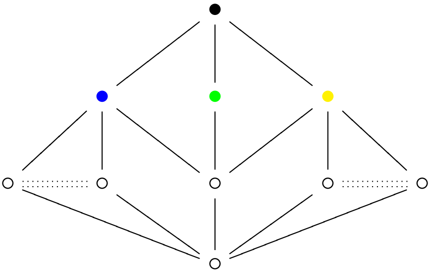

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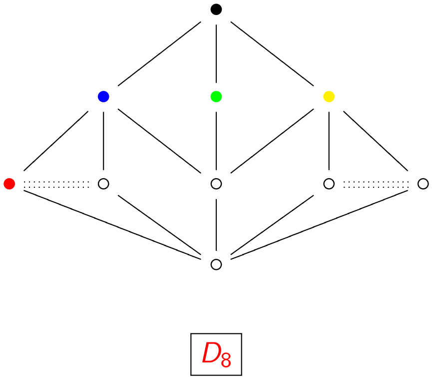

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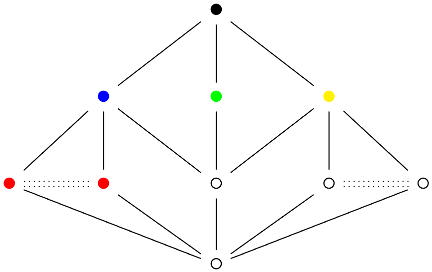

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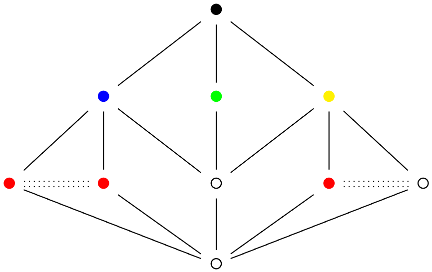

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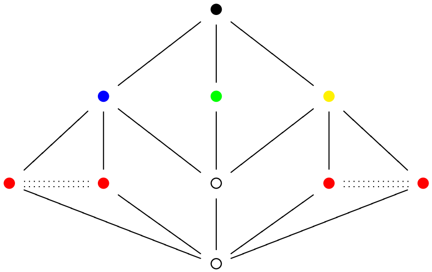

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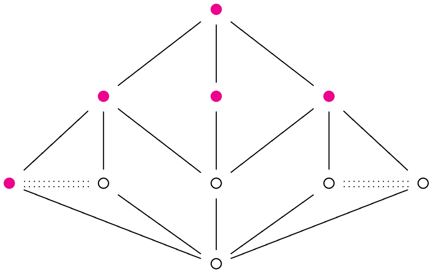

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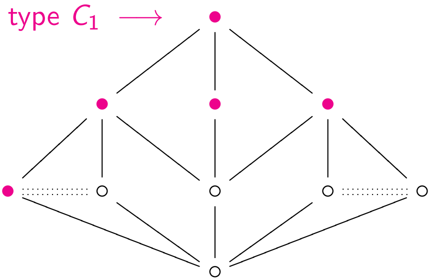

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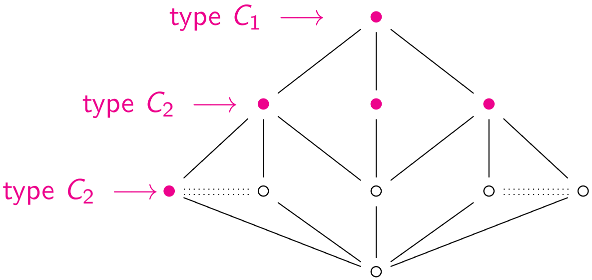

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is split injective, with explicit left inverse $\mathcal{D}_{\mathcal{B}}$ (i.e. $\mathcal{D}_{\mathcal{B}} \circ \mathcal{I}_{\mathcal{B}} = Id$).

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   is split injective, with explicit left inverse $\mathcal{D}_\mathcal{B}$ (i.e. $\mathcal{D}_\mathcal{B} \circ \mathcal{I}_\mathcal{B} = Id$).

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*The category $\mathcal{R}_p^\sharp$ is pre-additive, and the rational p-biset functors are the additive functors $\mathcal{C}_p \to \mathbb{Z}$-**Mod** which factor through $\mathcal{C}_p \to \mathcal{R}_p^\sharp$.*

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An example

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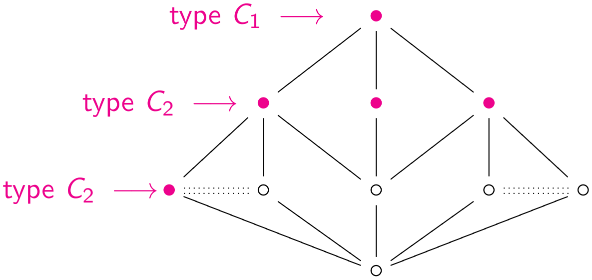

$\boxed{D_8}$

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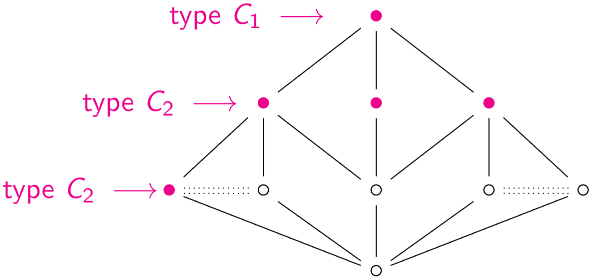

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$$D_8 \cong$$

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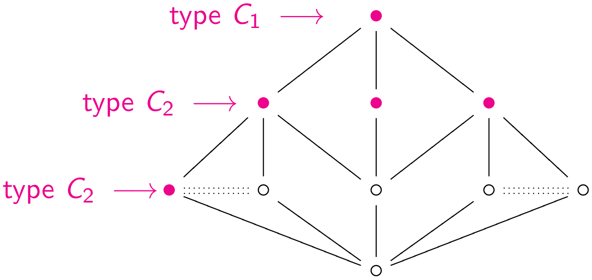

D_8

$$D_8 \cong \mathbf{1} \oplus 4 \cdot \partial C_2 .$$

Tensor structure

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Let P and Q be Roquette p-groups. Then

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where $\nu_{P,Q} \in \mathbb{N} - \{0\}$ *(explicit)*

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Let P and Q be Roquette p-groups. Then

$$\partial P \times \partial Q \cong \nu_{P,Q} \cdot \partial(P \diamond Q),$$

where $\nu_{P,Q} \in \mathbb{N} - \{0\}$ and $P \diamond Q$ is a Roquette group.

Examples :

- $\partial Q_8 \times \partial Q_8 \cong \partial C_2 \cong \mathbf{1}$.
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  - $\partial Q_8 \times \partial SD_{2^m} \cong \partial SD_{2^m}$, $m \geq 4$.
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