On restricted rational Cherednik algebras for exceptional complex reflection groups

Ulrich Thiel

(TU Kaiserslautern)

Partially supported by: DFG Schwerpunktprogramm Darstellungstheorie 1388

V: 1, Tuesday 20th March, 2012, 20:49

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- choose $0 \neq \alpha_s \in \mathrm{im}(\mathrm{id}_V - s)$.
- let $\alpha_s^\vee \in V^*$ be the unique element with $s(v) = v - \langle v, \alpha_s^\vee \rangle \alpha_s$ for all $v \in V$.

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Fact

For $R = K = \mathbb{C}$ the representation theory of the algebras H_c helps to understand the geometry of $(V \oplus V^*)/G$.

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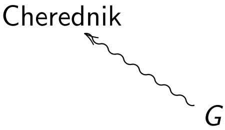

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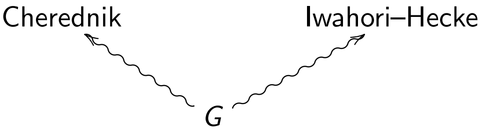

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Theorem (T., 2011)

Two simple KG-modules λ, μ lie in the same generic Euler block if and only if

$$\chi_\mu(1)\chi_\lambda(s) = \chi_\lambda(1)\chi_\mu(s)$$

for all $s \in \mathcal{S}$.

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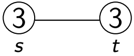
Generalized Coxeter diagram of G_4:



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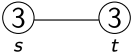

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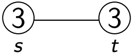

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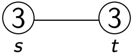

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Character table:

	id	s	s^2	...
$\phi_{1,0}$	1	1	1	...
$\phi_{1,4}$	1	ζ	ζ^2	...
$\phi_{1,8}$	1	ζ^2	ζ	...
$\phi_{2,5}$	2	-1	-1	...
$\phi_{2,3}$	2	$-\zeta$	$\zeta+1$	...
$\phi_{2,1}$	2	$\zeta+1$	$-\zeta$	...
$\phi_{3,2}$	3	0	0	...

$\zeta := \exp(\frac{2\pi i}{3})$.

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(a) All questions answered for G_4 for *all* parameters c.
(b) The $c : \mathcal{S}/G \to \mathbb{C}$ for which CM_c is non-trivial form a union of hyperplanes in $\mathbb{C}^2$.

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Let $c : \mathcal{S}/G \to R$ and let $\lambda \in \mathrm{Simp}(KG)$.

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(a) $\dim L_c(\lambda) \leq |G|$.
(b) If $\dim L_c(\lambda) < |G|$, then $L_c(\lambda)$ lies in a non-isolated CM-block.

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If $\dim L_c(\lambda) = |G|$, then

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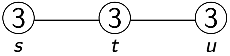
The exceptional group G_{25}

Generalized Coxeter diagram of G_{25}:



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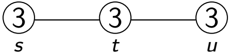

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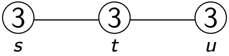

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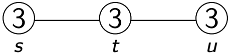

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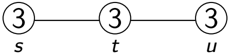

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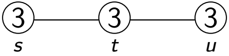

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Theorem (T., 2011)

Martino's conjecture is wrong!