Derived equivalence classification of cluster-tilted algebras of Dynkin type E

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Abstract

We address the question of when cluster-tilted algebras of Dynkin type E are derived equivalent and as main result obtain a complete derived equivalence classification. It turns out that two cluster-tilted algebras of type E are derived equivalent if and only if their Cartan matrices represent equivalent bilinear forms over the integers. For type E_6 all details are given in the paper, for types E_7 and E_8 we present the results in a concise form from which our findings should easily be verifiable.

1 Introduction

Cluster algebras have been introduced by Fomin and Zelevinsky around 2000 and have enjoyed a remarkable success story in recent years. They attractively link various areas of mathematics, like combinatorics, algebraic Lie theory, representation theory, algebraic geometry and integrable systems and have applications to mathematical physics. In an attempt to 'categorify' cluster algebras (without coefficients), cluster categories have been introduced by Buan, Marsh, Reiten, Reineke, Todorov [5]. More precisely, these are orbit categories of the form $\mathcal{C}_Q = D^b(KQ)/\tau^{-1}[1]$ where Q is a quiver without oriented cycles, $D^b(KQ)$ is the bounded derived category of the path algebra KQ (over an algebraically closed field K) and τ and [1] are the Auslander-Reiten translation and shift functor on $D^b(KQ)$, respectively. Remarkably, these cluster categories are again triangulated categories by a result of Keller [13].

Quivers of Dynkin types ADE play a special role in the theory of cluster algebras since they parametrize cluster-finite cluster algebras, by a seminal result of Fomin and Zelevinsky [10]. The corresponding cluster categories $\mathcal{C}_Q$ where Q is a Dynkin quiver are triangulated categories with finitely many indecomposable objects and their structure is well understood by work of Amiot [1].

Important objects in cluster categories are the cluster-tilting objects. A cluster-tilted algebra of type Q is by definition the endomorphism algebra of a cluster-tilting object in the cluster category $\mathcal{C}_Q$. The corresponding cluster-tilted algebras of Dynkin types A, D or E are of finite representation type and they can be constructed explicitly by quivers and relations. Namely, the quivers of the cluster-tilted algebras of Dynkin type Q are precisely the ones obtained from Q by performing finitely many quiver mutations. Moreover, in the Dynkin case, the quiver of a cluster-tilted algebra uniquely determines the relations [8]; we shall review the corresponding algorithm in Section 2 below.

In this paper we address the question of when two cluster-tilted algebras of Dynkin type E_6, E_7 or E_8 have equivalent derived categories.

The analogous question has been settled for cluster-tilted algebras of type A_n by Buan and Vatne [9] (see also work of Murphy on the more general case of m-cluster tilted algebras of type A_n [17]) and by Bastian [3] for type $\tilde{A}$. Note that the cluster-tilted algebras in these cases are gentle algebras [2].

It turns out that two cluster-tilted algebras of type A_n are derived equivalent if and only if their quivers have the same number of 3-cycles. For distinguishing such algebras up to derived equivalence one uses the determinants of the Cartan matrices; these have been determined explicitly for arbitrary gentle algebras by the second author in [12].

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A derived equivalence classification of cluster-tilted algebras of other Dynkin types D and E has been open. In this paper we settle this question for type E, i.e. we obtain a complete derived equivalence classification for cluster-tilted algebras of types E_6, E_7 and E_8. More precisely, our main result is the following.

Theorem 1.1. *Two cluster-tilted algebras of Dynkin type E are derived equivalent if and only if their Cartan matrices represent equivalent bilinear forms over $\mathbb{Z}$. This in turn happens if and only if the Cartan matrices of the algebras have the same determinant and the same characteristic polynomial of their asymmetry matrices.*

For the proof, we first need the possible quivers of the cluster-tilted algebras, i.e. the mutation class of a Dynkin quiver of type E; note that these mutation classes are finite. This will be achieved conveniently using Keller's software [14]. It suffices to get a list of representatives of the quivers modulo sink/source equivalence, since sink/source equivalent algebras will be derived equivalent. As the quivers determine the relations for cluster-tilted algebras of Dynkin type, we can compute the Cartan matrix of each of the cluster-tilted algebras of type E.

A natural strategy is first to divide these algebras into equivalence classes according to some invariants of derived equivalence, so that algebras belonging to different classes are not derived equivalent, and then to construct explicit tilting complexes for enough pairs within each class, thus proving that the algebras belonging to the same class are indeed derived equivalent.

The invariant of derived equivalence we use is the integer equivalence class of the bilinear form represented by the Cartan matrix of an algebra A. As this invariant is sometimes arithmetically subtle to compute directly, we instead compute the determinant of the Cartan matrix C_A and the characteristic polynomial of its asymmetry matrix $S_A = C_A C_A^{-T}$, defined whenever C_A is invertible over $\mathbb{Q}$, and encode them conveniently in a single polynomial that we call the *polynomial associated to* C_A. This quantity is generally a weaker invariant of derived equivalence, but in our case it will turn out to be enough for the classification. Note that unlike as in type A, the determinant itself is not sufficient for distinguishing the algebras up to derived equivalence.

We stress that the asymmetry matrix and its characteristic polynomial are well defined whenever the Cartan matrix is invertible over $\mathbb{Q}$, even without having any categorical meaning, as follows from [16, Section 3.3]. In the special case when A has finite global dimension, the asymmetry matrix S_A, or better minus its transpose $-C_A^{-1}C_A^T$, is related to the Coxeter transformation which does carry categorical meaning, and its characteristic polynomial is known as the Coxeter polynomial of the algebra.

For those algebras having the same Cartan determinant and the same characteristic polynomial of the asymmetry matrix we then construct explicit tilting complexes in order to prove them to be derived equivalent. This forms the main body of the technical work involved to achieve the derived equivalence classification.

Let us briefly describe the above main result in some more detail. For precise definitions of the cluster-tilted algebras involved we refer to Sections 3 (type E_6), A/B (type E_7), and C/D (type E_8) below.

For type E_6 there are 21 cluster-tilted algebras, up to sink/source equivalence. They turn out to fall into six derived equivalence classes. These six classes are characterized by the following Cartan determinants and characteristic polynomial of the asymmetry matrix, respectively.

Derived equivalence classes for type E_6			
$\det C_A$	characteristic polynomial of S_A	$\det C_A$	characteristic polynomial of S_A
1	$x^6 - x^5 + x^3 - x + 1$	3	$x^6 + x^3 + 1$
2	$x^6 - x^4 + 2x^3 - x^2 + 1$	4	$x^6 + x^4 + x^2 + 1$
2	$x^6 - 2x^4 + 4x^3 - 2x^2 + 1$	4	$x^6 + x^5 - x^4 + 2x^3 - x^2 + x + 1$

For type E_7 the mutation class consists of 112 quivers up to sink/source equivalence. The derived equivalence classes of the cluster-tilted algebras are again characterized by the Cartan determinant and the characteristic polynomial of the asymmetry matrix; there are 14 classes in total, given as follows.

Derived equivalence classes for type E_7			
$\det C_A$	characteristic polynomial of S_A	$\det C_A$	characteristic polynomial of S_A
1	$x^7 - x^6 + x^4 - x^3 + x - 1$	4	$x^7 + x^6 - 2x^5 + 2x^4 - 2x^3 + 2x^2 - x - 1$
2	$x^7 - x^5 + 2x^4 - 2x^3 + x^2 - 1$	4	$x^7 + x^5 - x^4 + x^3 - x^2 - 1$
2	$x^7 - x^5 + x^4 - x^3 + x^2 - 1$	4	$x^7 + x^5 - 2x^4 + 2x^3 - x^2 - 1$
2	$x^7 - 2x^5 + 4x^4 - 4x^3 + 2x^2 - 1$	5	$x^7 + x^5 - x^4 + x^3 - x^2 - 1$
3	$x^7 - 1$	6	$x^7 + x^6 - x^4 + x^3 - x - 1$
4	$x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x - 1$	6	$x^7 + x^5 - x^2 - 1$
4	$x^7 + x^6 - x^5 - x^4 + x^3 + x^2 - x - 1$	8	$x^7 + x^6 + x^5 - x^4 + x^3 - x^2 - x - 1$

For type E_8 we get 391 algebras, up to sink/source equivalence. They turn out to fall into 15 different derived equivalence classes which are characterized as follows.

Derived equivalence classes for type E_8			
$\det C_A$	characteristic polynomial of S_A	$\det C_A$	characteristic polynomial of S_A
1	$x^8 - x^7 + x^5 - x^4 + x^3 - x + 1$	4	$x^8 + x^6 - x^5 + 2x^4 - x^3 + x^2 + 1$
2	$x^8 - x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 + 1$	4	$x^8 + x^6 - 2x^5 + 4x^4 - 2x^3 + x^2 + 1$
2	$x^8 - x^6 + x^5 + x^3 - x^2 + 1$	5	$x^8 + x^6 + x^4 + x^2 + 1$
2	$x^8 - 2x^6 + 4x^5 - 4x^4 + 4x^3 - 2x^2 + 1$	6	$x^8 + x^6 + x^5 + x^3 + x^2 + 1$
3	$x^8 + x^4 + 1$	6	$x^8 + x^7 + 2x^4 + x + 1$
4	$x^8 + x^7 - x^6 + x^5 + x^3 - x^2 + x + 1$	8	$x^8 + 2x^7 + 2x^4 + 2x + 1$
4	$x^8 + x^7 - x^6 + 2x^4 - x^2 + x + 1$	8	$x^8 + x^7 + x^6 + 2x^4 + x^2 + x + 1$
4	$x^8 + x^7 - 2x^6 + 2x^5 + 2x^3 - 2x^2 + x + 1$		

The paper is organized as follows. In Section 2 we collect some background material; in particular we recall the fundamental notion of quiver mutation, describe the results of Buan, Marsh and Reiten on cluster-tilted algebras of finite representation type, review the fundamental results on derived equivalences and then discuss invariants of derived equivalence such as the equivalence class of the Euler form, in particular leading to the determinant of the Cartan matrix and the characteristic polynomial of its asymmetry matrix as derived invariants.

In Section 3 we discuss derived equivalences for cluster-tilted algebras of Dynkin type E_6 in detail. We first list the mutation class of an E_6 quiver, up to sink/source equivalence; this list has been produced using Keller's software and comprises 21 algebras. We also give the corresponding Cartan matrices and compute their determinants and the characteristic polynomials of their asymmetry matrices. The main result of this section is Theorem 3.1 which proves the main Theorem 1.1 for type E_6. For this we have to find explicit tilting complexes for the cluster-tilted algebras of type E_6 and we have to determine their endomorphism rings. The necessary calculations are carried out and described in detail.

For types E_7 and E_8 we have followed a different strategy of presentation since the number of algebras involved becomes very large. We first list the algebras but without drawing the quivers; again, the quivers have been found using Keller's software. We then present the results on derived equivalences for cluster-tilted algebras of types E_7 and E_8 in a very concise form which is explained at the beginning of the respective sections. For each group of algebras with the same Cartan determinant and characteristic polynomial of the asymmetry matrix we then provide tilting complexes and list their endomorphism rings, but without giving any details on the calculations. However, we hope that we have provided enough information so that interested readers should easily be able to check our findings.

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2 Preliminaries

2.1 Quiver mutations

A *quiver* is a finite directed graph Q, consisting of a finite set of vertices Q_0 and a finite set of arrows Q_1 between them. A fundamental concept in the theory of Fomin and Zelevinsky's cluster algebras is mutation; for quivers this takes the following shape.

Definition 2.1. Let Q be a quiver without loops and oriented 2-cycles. For vertices i, j, let a_{ij} denote the number of arrows from i to j, where $a_{ij} < 0$ means that there are $-a_{ij}$ arrows from j to i.

The *mutation of* Q *at the vertex* k yields a new quiver Q' obtained from Q by the following procedure:

1. Add a new vertex k^*.
2. For all vertices $i \neq j$, different from k, such that $a_{ij} \geq 0$, set the number of arrows a'_{ij} from i to j in Q' as follows:

   if $a_{ik} \geq 0$ and $a_{kj} \geq 0$, then $a'_{ij} := a_{ij} + a_{ik}a_{kj}$;

   if $a_{ik} \leq 0$ and $a_{kj} \leq 0$, then $a'_{ij} := a_{ij} - a_{ik}a_{kj}$.
3. For any vertex i, replace all arrows from i to k with arrows from k^* to i, and replace all arrows from k to i with arrows from i to k^*.
4. Remove the vertex k.

Two quivers are called *mutation-equivalent* if one can be obtained from the other by a finite sequence of mutations. The *mutation class* of a quiver Q is the class of all quivers mutation-equivalent to Q. It is known from the seminal results of Fomin and Zelevinsky [10] that the mutation class of a Dynkin quiver Q is finite.

2.2 Cluster-tilted algebras of finite representation type

Cluster-tilted algebras arise as endomorphism algebras of cluster-tilting objects in a cluster category, see [6]. For the special case of Dynkin quivers the cluster-tilted algebras are known to be of finite representation type. Moreover, by a result of Buan and Reiten [8] they can be described as quivers with relations by a simple combinatorial recipe to be recalled below. As a consequence, a cluster-tilted algebra of Dynkin type is uniquely determined by its quiver.

Let Q be a quiver and throughout this paper let K be an algebraically closed field. We can form the *path algebra* KQ, where the basis of KQ is given by all paths in Q, including trivial paths e_i of length zero at each vertex i of Q. Multiplication in KQ is defined by concatenation of paths. Our convention is to compose paths from right to left. For any path α in Q let $s(\alpha)$ denote its start vertex and $t(\alpha)$ its end vertex. Then the product of two paths α and β is defined to be the concatenated path $\alpha\beta$ if $s(\alpha) = t(\beta)$. The unit element of KQ is the sum of all trivial paths, i.e., $1_{KQ} = \sum\limits_{i \in Q_0} e_i$.

We recall some background from [8]. An oriented cycle in a quiver is called *full* if it does not contain any repeated vertices and if the subquiver generated by the cycle contains no other arrows. If there is an arrow $i \to j$ in a quiver Q then a path from j to i is called *shortest path* if the induced subquiver is a full cycle.

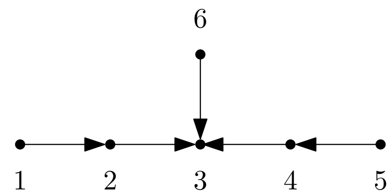
We now describe cluster-tilted algebras of Dynkin type by a quiver with relations, i.e. in the form KQ/I where Q is a finite quiver and I is some admissible ideal in the path algebra KQ. Recall that the quivers associated with cluster-tilted algebras of Dynkin type are precisely the quivers in the the mutation class of the corresponding Dynkin quiver.

Relations are linear combinations $k_1\omega_1 + \cdots + k_m\omega_m$ of paths ω_i in Q, all starting in the same vertex and ending in the same vertex, and with each k_i non-zero in K. If $m = 1$, we call the relation a *zero-relation.* If $m = 2$ and $k_1 = 1,\ k_2 = -1$, and we call it a *commutativity-relation* (and say that the paths ω_1 and ω_2 commute). It will turn out that for cluster-tilted algebras of Dynkin type the ideal I can be generated by only using zero-relations and commutativity relations. Finally, a relation ρ is called *minimal* if whenever $\rho = \sum_i \beta_i \circ \rho_i \circ \gamma_i$, where ρ_i is a relation for every i, then there is an index j such that both β_j and γ_j are scalars.

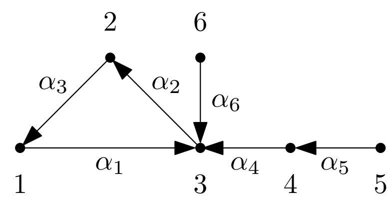
Proposition 2.2 (Buan and Reiten [8]). *A cluster-tilted algebra A of finite representation type is of the form $A = KQ/I$, where Q is mutation equivalent to a Dynkin quiver and where the ideal I can be described as follows. Let i and j be vertices in Q.*

1. *The ideal I is generated by minimal zero-relations and minimal commutativity-relations.*
2. *Assume there is an arrow $i \to j$. Then there are at most two shortest paths from j to i.*
   - *i) If there is exactly one, then this is a minimal zero-relation.*
   - *ii) If there are two, ω and μ, then ω and μ are not zero in A and there is a minimal relation $\omega - \mu$.*
3. *Up to multiplication by non-zero elements of K there are no other minimal zero-relations or commutativity-relations than the ones coming from 2.*

Example 2.3. We consider the following quiver Q of type E_6

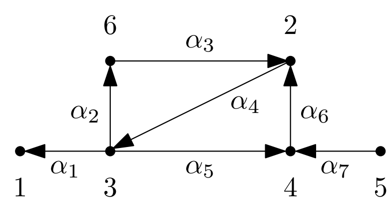


If we mutate at vertex 2, we get the following quiver Q'



The corresponding cluster-tilted algebra is of the form $A = KQ'/I$ where I is generated by the zero-relations $\alpha_1\alpha_3,\ \alpha_2\alpha_1$ and $\alpha_3\alpha_2$ (and there are no commutativity-relations).

Mutating the latter quiver at the vertex 3 leads to the quiver Q''



Here, the ideal of relations of the corresponding cluster-tilted algebra is generated by the zero-relations $\alpha_2\alpha_4$, $\alpha_5\alpha_4$, $\alpha_4\alpha_3$ and $\alpha_4\alpha_6$ and the commutativity-relation $\alpha_3\alpha_2 = \alpha_6\alpha_5$.

2.3 Tilting complexes and derived equivalences

In this section we briefly review the fundamental results on derived equivalences. All algebras are assumed to be finite-dimensional K-algebras.

For a K-algebra A the bounded derived category of A-modules is denoted by $D^b(A)$. Recall that two algebras A, B are called derived equivalent if $D^b(A)$ and $D^b(B)$ are equivalent as triangulated categories. By a famous theorem of Rickard [18] derived equivalences can be found using the concept of tilting complexes.

Definition 2.4. A *tilting complex* T over A is a bounded complex of finitely generated projective A-modules satisfying the following conditions:

i) $\mathrm{Hom}_{D^b(A)}(T, T[i]) = 0$ for all $i \neq 0$, where $[1]$ denotes the shift functor in $D^b(A)$;

ii) the category $\mathrm{add}(T)$ (i.e. the full subcategory consisting of direct summands of direct sums of T) generates the homotopy category $K^b(P_A)$ of projective A-modules as a triangulated category.

We can now formulate Rickard's seminal result.

Theorem 2.5 (Rickard [18])**.** *Two algebras A and B are derived equivalent if and only if there exists a tilting complex T for A such that the endomorphism algebra $\mathrm{End}_{D^b(A)}(T) \cong B$.*

2.4 The equivalence class of the Euler form as derived invariant

Let A be a finite-dimensional algebra over a field K and let $P_1, \dots, P_n$ be a complete collection of non-isomorphic indecomposable projective A-modules (finite-dimensional over K). The *Cartan matrix* of A is then the $n \times n$ matrix C_A defined by $(C_A)_{ij} = \dim_K \mathrm{Hom}(P_j, P_i)$.

Denote by $\mathrm{per}\, A$ the triangulated category of *perfect* complexes of A-modules inside the derived category of A, that is, complexes (quasi-isomorphic) to finite complexes of finitely generated projective A-modules. The Grothendieck group $K_0(\mathrm{per}\, A)$ is a free abelian group on the generators $[P_1], \dots, [P_n]$, and the expression

$$\langle X, Y \rangle = \sum_{r \in \mathbb{Z}} (-1)^r \dim_K \mathrm{Hom}_{\mathrm{per}\, A}(X, Y[r])$$

is well defined for any $X, Y \in \mathrm{per}\, A$ and induces a bilinear form on $K_0(\mathrm{per}\, A)$, known as the *Euler form*, whose matrix with respect to the basis of projectives is C_A^T.

The following proposition is well known. For the convenience of the reader, we give the short proof, see also the *proof* of Proposition 1.5 in [4].

Proposition 2.6. *Let A and B be two finite-dimensional, derived equivalent algebras. Let n denote by number of their non-isomorphic indecomposable projectives. Then the matrices C_A and C_B represent equivalent bilinear forms over $\mathbb{Z}$, that is, there exists $P \in \mathrm{GL}_n(\mathbb{Z})$ such that $PC_AP^T = C_B$.*

Proof. Indeed, by [18], if A and B are derived equivalent, then $\mathrm{per}\, A$ and $\mathrm{per}\, B$ are equivalent as triangulated categories. Now any triangulated functor $F : \mathrm{per}\, A \to \mathrm{per}\, B$ induces a linear map from $K_0(\mathrm{per}\, A)$ to $K_0(\mathrm{per}\, B)$. When F is also an equivalence, this map is an isomorphism of the Grothendieck groups preserving the Euler forms. Thus, if $[F]$ denotes the matrix of this map with respect to the bases of indecomposable projectives, then $[F]^T C_B [F] = C_A$. □

In general, to decide whether two integral bilinear forms are equivalent is a very subtle arithmetical problem. Therefore, it is useful to introduce somewhat weaker invariants that are computationally easier to handle. In order to do this, assume further that C_A is invertible over $\mathbb{Q}$. In this case one can consider the rational matrix $S_A = C_A C_A^{-T}$ (here C_A^{-T} denotes the inverse of the transpose of C_A), known in the theory of non-symmetric bilinear forms as the *asymmetry* of C_A.

Proposition 2.7. *Let A and B be two finite-dimensional, derived equivalent algebras with invertible (over $\mathbb{Q}$) Cartan matrices. Then we have the following assertions, each implied by the preceding one:*

1. *There exists $P \in \mathrm{GL}_n(\mathbb{Z})$ such that $PC_AP^T = C_B$.*
2. *There exists $P \in \mathrm{GL}_n(\mathbb{Z})$ such that $PS_AP^{-1} = S_B$.*
3. *There exists $P \in \mathrm{GL}_n(\mathbb{Q})$ such that $PS_AP^{-1} = S_B$.*
4. *The matrices S_A and S_B have the same characteristic polynomial.*

For proofs and discussion, see for example [16, Section 3.3]. Since the determinant of an integral bilinear form is invariant under equivalence, we can combine it with the characteristic polynomial $p_{S_A}(x)$ of the asymmetry matrix S_A to obtain a discrete invariant of derived equivalence, namely $(\det C_A) \cdot p_{S_A}(x)$. We call this invariant the *polynomial associated with C_A*.

Remark 2.8. The matrix $S_A = C_A C_A^{-T}$ (or better, minus its transpose $-C_A^{-1}C_A^T$) is related to the *Coxeter transformation* which has been widely studied in the case when A has finite global dimension (so that C_A is invertible over $\mathbb{Z}$). It is the K-theoretic shadow of the Serre functor and the related Auslander-Reiten translation in the derived category. The characteristic polynomial is then known as the *Coxeter polynomial* of the algebra.

Remark 2.9. In general, S_A might have non-integral entries. However, when the algebra A is *Gorenstein*, the matrix S_A is integral, which is an incarnation of the fact that the injective modules have finite projective resolutions. By a result of Keller and Reiten [15], this is the case for the cluster-tilted algebras in question.

2.5 Computations of Cartan matrices

Let $A = KQ/I$ be an algebra given by a quiver $Q = (Q_0, Q_1)$ with relations. Since $\sum_{i\in Q_0} e_i$ is the unit element in A we get a decomposition $A = A \cdot 1 = \bigoplus_{i\in Q_0} Ae_i$, hence the (left) A-modules $P_i := Ae_i$ are the indecomposable projective A-modules, and the Cartan matrix $C_A = (c_{ij})$ of A is the n-by-n matrix whose entries are $c_{ij} = \dim_K \operatorname{Hom}_A(P_j, P_i)$, where $n = |Q_0|$. Any homomorphism $\varphi : Ae_j \to Ae_i$ of left A-modules is uniquely determined by $\varphi(e_j) \in e_j A e_i$, the K-vector space generated by all paths in Q from vertex i to vertex j that are nonzero in A. In particular, we have $c_{ij} = \dim_K e_j A e_i$, i.e., computing entries of the Cartan matrix for A reduces to counting paths in Q.

For cluster-tilted algebras of Dynkin type the entries of the Cartan matrix can only be 0 or 1, as the following result shows.

Proposition 2.10 (Buan, Marsh, Reiten [7])**.** *Let A be a cluster-tilted algebra of finite representation type. Then $\dim_K \operatorname{Hom}_A(P_j, P_i) \leq 1$ for any two indecomposable projective A-modules P_i and P_j.*

Example 2.11. We have a look at the quivers in Example 2.3 again, and compute the Cartan matrices of the corresponding cluster-tilted algebras.

For the Dynkin quiver Q of type E_6 with the above orientation we get the following Cartan matrix $\begin{pmatrix} 1&1&1&0&0&0\\ 0&1&1&0&0&0\\ 0&0&1&0&0&0\\ 0&0&1&1&0&0\\ 0&0&1&1&1&0\\ 0&0&1&0&0&1 \end{pmatrix}$ since there are no zero- or commutativity-relations.

For the quiver Q' obtained by mutation from Q at vertex 2, the corresponding Cartan matrix C' has the form $C' = \begin{pmatrix} 1&0&1&0&0&0\\ 1&1&0&0&0&0\\ 0&1&1&0&0&0\\ 0&1&1&1&0&0\\ 0&1&1&1&1&0\\ 0&1&1&0&0&1 \end{pmatrix}$ for KQ/I since the paths from vertex 1 to 2, from 2 to 3 and from 3 to 1 are zero.

Finally, for the quiver Q'' obtained from Q' by mutating at vertex 3, the cluster-tilted algebra has Cartan matrix $C'' = \begin{pmatrix} 1&0&0&0&0&0\\ 1&1&1&0&0&0\\ 1&1&1&1&0&1\\ 0&1&0&1&0&0\\ 0&1&0&1&1&0\\ 0&1&0&0&0&1 \end{pmatrix}$. Note that the two paths from vertex 3 to vertex 2 (over 4 or 6) are the same since we have the commutativity-relation $\alpha_3\alpha_2 = \alpha_6\alpha_5$.

For calculating the endomorphism ring $\operatorname{End}_{D^b(A)}(T)$ of a tilting complex T over the algebra A, we can use the following statement which explicitly gives the Cartan matrix of the endomorphism ring in terms of the tilting complex and the Cartan matrix of A.

Proposition 2.12. *Let T be a tilting complex over A with endomorphism algebra $B = \operatorname{End}_{D^b(A)}(T)$, and let $T_1, \ldots, T_n$ be the indecomposable direct summands of T.*

Then the Cartan matrix C_B of B is given by $C_B = PC_AP^T$, where $P = (p_{ij})_{i,j=1}^n$ is the matrix defined by

$$[T_i] = \sum_{j=1}^{n} p_{ij}[P_j]$$

(that is, its i-th row is the class of the summand T_i in $K_0(\operatorname{per} A)$ written in the basis $[P_1], \dots, [P_n]$).

Example 2.13. Continuing Example 2.11, let $T = T_1 \oplus \cdots \oplus T_6$ be the complex over the cluster-tilted algebra corresponding to Q' defined by

$$T_i = \begin{cases} P_i & \text{if } i \neq 3 \\ P_3 \to P_1 \oplus P_4 \oplus P_6 & \text{if } i = 3, \end{cases}$$

where the P_i are in degree 0 for $i \neq 3$ and P_3 is in degree -1.

Then T is a tilting complex and the corresponding matrix P is given by

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

so that $C'' = PC'P^T$. In fact, $\operatorname{End} T$ is isomorphic to the cluster-tilted algebra corresponding to Q'', see Section 3.3.1.

It is sometimes convenient to use the following alternating sum formula, arising from the fact that for a bounded complex $X = (X^r)$ of projective modules, we have $[X] = \sum (-1)^r [X^r]$ in $K_0(\operatorname{per} A)$.

Proposition 2.14 (Happel [11])**.** *For an algebra A let $X = (X^r)_{r \in \mathbb{Z}}$ and $Y = (Y^s)_{s \in \mathbb{Z}}$ be bounded complexes of projective A-modules. Then*

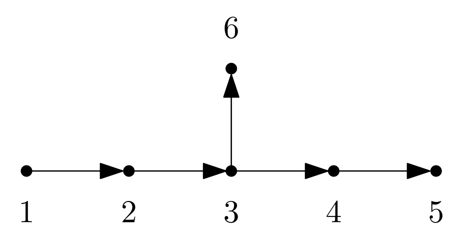
$$\sum_i (-1)^i \dim \operatorname{Hom}_{K^b(P_A)}(X, Y[i]) = \sum_{r,s} (-1)^{r-s} \dim \operatorname{Hom}_A(X^r, Y^s).$$

In particular, if X and Y are direct summands of the same tilting complex, then

$$\dim \operatorname{Hom}_{K^b(P_A)}(X, Y) = \sum_{r,s} (-1)^{r-s} \dim \operatorname{Hom}_A(X^r, Y^s).$$

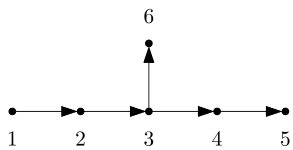
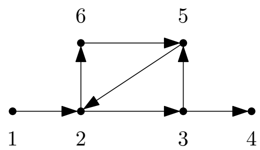
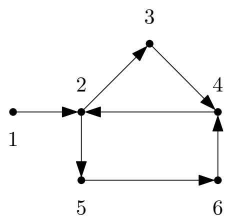
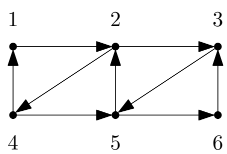
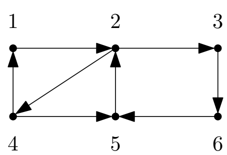
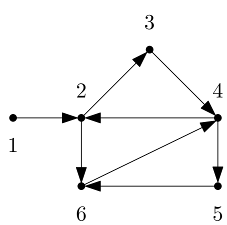
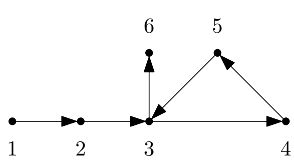
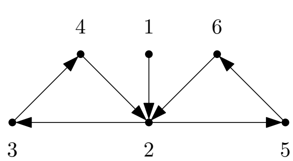
3 Derived equivalences of cluster-tilted algebras of type E_6

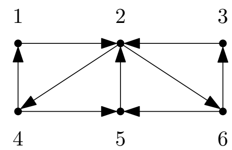
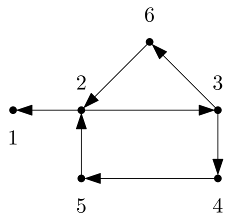
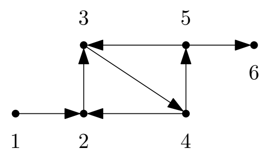
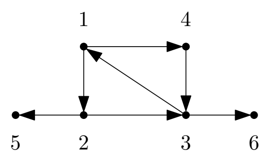
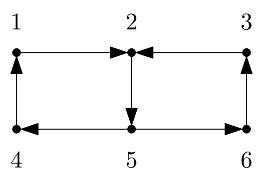
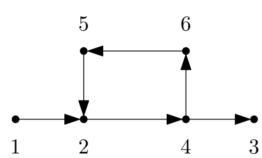
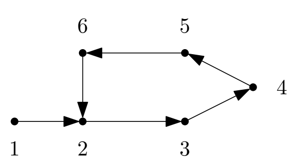
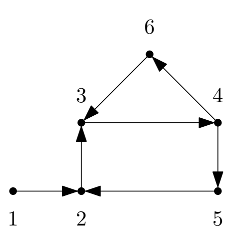
For the mutation class of E_6 we start with the following quiver

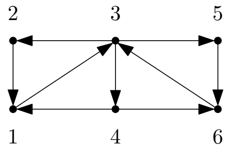
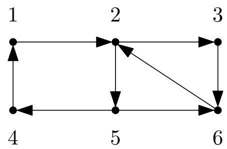
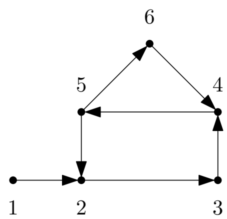
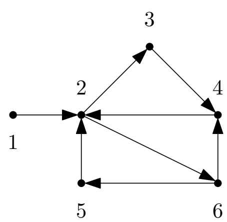
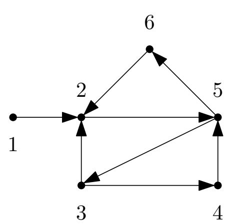


and compute all quivers which can be obtained from it by a finite number of mutations. For this, we used the software of B. Keller [14]. The class we get consists (up to sink/source equivalence) of 21 different algebras. We can divide these algebras into 6 groups by computing the polynomials associated with their Cartan matrices. Recall that these are obtained by multiplying the determinant of the Cartan matrix by the characteristic polynomial of its asymmetry matrix.

We list, for each of the 21 cluster-tilted algebras A_i of type E_6 (up to sink/source equivalence), its quiver, its Cartan matrix C_{A_i} and the associated polynomial.

no.	quiver Q	Cartan matrix	polynomial
1		$\begin{pmatrix} 1&1&1&1&1&1\\ 0&1&1&1&1&1\\ 0&0&1&1&1&1\\ 0&0&0&1&1&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	$x^6 - x^5 + x^3 - x + 1$
2		$\begin{pmatrix} 1&1&1&1&1&1\\ 0&1&1&1&1&1\\ 0&0&1&1&1&0\\ 0&0&0&1&0&0\\ 0&1&0&0&1&0\\ 0&0&0&0&1&1 \end{pmatrix}$	$2(x^6 - x^4 + 2x^3 - x^2 + 1)$
3		$\begin{pmatrix} 1&1&1&1&1&1\\ 0&1&1&1&1&1\\ 0&0&1&1&0&0\\ 0&1&0&1&1&0\\ 0&0&0&1&1&1\\ 0&1&0&1&0&1 \end{pmatrix}$	$3(x^6 + x^3 + 1)$
4		$\begin{pmatrix} 1&1&1&0&0&0\\ 0&1&1&1&1&0\\ 0&0&1&0&1&0\\ 1&1&1&1&1&1\\ 0&1&1&0&1&1\\ 0&0&1&0&0&1 \end{pmatrix}$	$3(x^6 + x^3 + 1)$
5		$\begin{pmatrix} 1&1&1&0&0&1\\ 0&1&1&1&1&1\\ 0&0&1&0&1&1\\ 1&1&1&1&1&0\\ 0&1&1&0&1&0\\ 0&1&0&0&1&1 \end{pmatrix}$	$4(x^6 + x^4 + x^2 + 1)$
6		$\begin{pmatrix} 1&1&1&1&0&1\\ 0&1&1&1&0&1\\ 0&0&1&1&1&0\\ 0&1&0&1&1&1\\ 0&0&0&0&1&1\\ 0&0&0&1&0&1 \end{pmatrix}$	$3(x^6 + x^3 + 1)$
7		$\begin{pmatrix} 1&1&1&1&0&1\\ 0&1&1&1&0&1\\ 0&0&1&1&0&1\\ 0&0&0&1&1&0\\ 0&0&1&0&1&1\\ 0&0&0&0&0&1 \end{pmatrix}$	$2(x^6 - x^4 + 2x^3 - x^2 + 1)$
8		$\begin{pmatrix} 1&1&1&0&1&0\\ 0&1&1&0&1&0\\ 0&0&1&1&0&0\\ 0&1&0&1&1&0\\ 0&0&0&0&1&1\\ 0&1&1&0&0&1 \end{pmatrix}$	$4(x^6 + x^5 - x^4 + 2x^3 - x^2 + x + 1)$

no.	quiver Q	Cartan matrix	polynomial
9		$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$	$4(x^6 + x^4 + x^2 + 1)$
10		$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	$3(x^6 + x^3 + 1)$
11		$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$2(x^6 - 2x^4 + 4x^3 - 2x^2 + 1)$
12		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$2(x^6 - x^4 + 2x^3 - x^2 + 1)$
13		$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	$4(x^6 + x^4 + x^2 + 1)$
14		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$	$3(x^6 + x^3 + 1)$
15		$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$	$4(x^6 + x^4 + x^2 + 1)$
16		$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	$3(x^6 + x^3 + 1)$

no.	quiver Q	Cartan matrix	polynomial
17		$\begin{pmatrix} 1&0&1&0&1&0\\ 1&1&0&0&0&0\\ 1&1&1&1&1&1\\ 1&0&1&1&0&1\\ 0&0&0&0&1&1\\ 0&1&1&0&0&1 \end{pmatrix}$	$4(x^6+x^4+x^2+1)$
18		$\begin{pmatrix} 1&1&1&0&1&1\\ 0&1&1&1&1&1\\ 0&0&1&0&0&1\\ 1&1&1&1&0&0\\ 1&1&0&1&1&1\\ 0&1&0&0&0&1 \end{pmatrix}$	$4(x^6+x^4+x^2+1)$
19		$\begin{pmatrix} 1&1&1&1&0&0\\ 0&1&1&1&0&0\\ 0&0&1&1&1&0\\ 0&1&0&1&1&0\\ 0&1&1&1&1&1\\ 0&0&0&1&0&1 \end{pmatrix}$	$3(x^6+x^3+1)$
20		$\begin{pmatrix} 1&1&1&1&0&1\\ 0&1&1&1&0&1\\ 0&0&1&1&0&0\\ 0&1&0&1&0&0\\ 0&1&1&0&1&0\\ 0&1&0&1&1&1 \end{pmatrix}$	$3(x^6+x^3+1)$
21		$\begin{pmatrix} 1&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&1&1&1&1&0\\ 0&0&0&1&1&1\\ 0&1&1&0&1&1\\ 0&1&0&0&0&1 \end{pmatrix}$	$3(x^6+x^3+1)$

The associated cluster-tilted algebras are denoted by A_{number}.

The following theorem is the main result of this section.

Theorem 3.1. *Two cluster-tilted algebras of type E_6 are derived equivalent if and only if their Cartan matrices represent equivalent bilinear forms over $\mathbb{Z}$.*

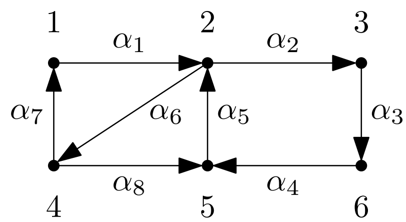
For proving the theorem we now have to show that the cluster-tilted algebras with the same Cartan determinant and the same characteristic polynomial are indeed derived equivalent. To this end, we shall explicitly construct suitable tilting complexes and determine their endomorphism algebras. Note that the class of cluster-tilted algebras is not closed under derived equivalences, so one carefully has to choose suitable tilting complexes in order to get another cluster-tilted algebras as endomorphism algebra.

3.1 Derived equivalences for polynomial $4(x^6 + x^4 + x^2 + 1)$

Since we deal with left modules and read paths from right to left, a nonzero path from vertex i to j gives a homomorphism $P_j \to P_i$ by right multiplication. Thus, two arrows $\alpha : i \to j$ and $\beta : j \to k$ give a path $\beta\alpha$ from i to k and a homomorphism $\alpha\beta : P_k \to P_i$.

3.1.1 A_5 is derived equivalent to A_9

First consider A_5 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_5-modules, where $T_i : 0 \to P_i \to 0,\ i \in \{1, 2, 4, 5, 6\}$, are complexes concentrated in degree zero and $T_3 : 0 \to P_3 \xrightarrow{\alpha_2} P_2 \to 0$ is a complex in degrees -1 and 0.

Now we want to show that T is a tilting complex. Property $i)$ of Definition 2.4 is clear for $|i| \geq 2$ since T is concentrated in two degrees.
We begin with possible maps $T_3 \to T_3[1]$ and $T_3 \to T_3[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_3 & \xrightarrow{\alpha_2} & P_2 & \to & 0 \\
 & & & & \downarrow \alpha_2 & & & & \\
0 & \to & P_3 & \xrightarrow{\alpha_2} & P_2 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_3 & \xrightarrow{\alpha_2} & P_2 & \to & 0
\end{array}$$

Here α_2 is a basis of the space of homomorphisms between P_3 and P_2.
But the homomorphism α_2 is homotopic to zero and in the second case there is no non-zero homomorphism $P_2 \to P_3$ (as we can see in the Cartan matrix of A_5).

Now let $i = -1$ and consider possible maps $T_3 \to T_j[-1],\ j \neq 3$. These maps are given by a map of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_3 & \xrightarrow{\alpha_2} & P_2 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_1, P_2, P_4, P_5 or direct sums of these.
Note that there is no non-zero homomorphism $P_3 \to P_6$ since this is a zero-relation in the quiver of A_5.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every path from vertex $i \in \{1, 2, 4, 5\}$ to vertex 3 ends with α_2. Hence, every homomorphism from P_3 to P_1, P_2, P_4 or P_5 starts with α_2, up to scalars and thus, every homomorphism $P_3 \to Q$ can be factored through the map $\alpha_2 : P_3 \to P_2$.
Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 4, 5, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, let $i = 1$. We have to consider maps $T_j \to T_3[1]$ for $j \neq 3$. These are given as follows

$$\begin{array}{ccccccc}
0 & \to & Q & \to & 0 & & \\
 & & \downarrow & & & & \\
0 & \to & P_3 & \xrightarrow{\alpha_2} & P_2 & \to & 0
\end{array}$$

where Q can be either P_5, P_6 or direct sums of these.
Note that $\mathrm{Hom}(P_j, P_3) = 0$ for $j = 1,\ 2$ and $j = 4$.
But no non-zero map can be zero when composed with α_2 since the path $\alpha_4\alpha_3\alpha_2 = \alpha_8\alpha_6 \neq 0$. So the only homomorphism of complexes $T_j \to T_3[1],\ j \neq 3$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_5})}(T, T[i]) = 0$ in the homotopy category.

Secondly we have to show that $\mathrm{add}(T)$ generates $\mathrm{K}^b(P_A)$ as a triangulated category. It suffices to show that the projective indecomposable modules $P_1, \dots, P_6$, viewed as stalk complexes, can be generated by

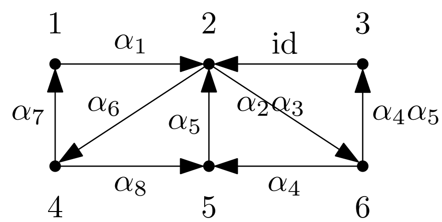
add(T). We denote by $P_k[n]$ the complex with P_k concentrated in degree n. Since P_k, $k \in \{1,2,4,5,6\}$, occur as summands of T, $P_k[0]$ is in add(T) for all $k \in \{1,2,4,5,6\}$ and thus $P_k[n]$ is in the triangulated category generated by add(T) for all $k \in \{1,2,4,5,6\}$ and for all n. Thus, we have to check that $P_3[n]$ can be generated by add(T).
There exists a homomorphism of complexes f from $P_2[0]$ to the complex $T_3 : 0 \to P_3 \xrightarrow{\alpha_2} P_2 \to 0$ given by id_{P_2} in degree zero. Then the stalk complex $P_3[1]$ can be shown to be homotopy equivalent (i.e., isomorphic in $\mathrm{K}^b(P_A)$) to the mapping cone $M(f) : 0 \to P_2 \oplus P_3 \xrightarrow{(\mathrm{id},\alpha_2)} P_2 \to 0$ of f. Thus, we have a distinguished triangle

$$\underbrace{P_2[0]}_{\in \mathrm{add}(T)} \xrightarrow{f} \underbrace{T_3}_{\in \mathrm{add}(T)} \to P_3[1] \to \underbrace{P_2[1]}_{\in \mathrm{add}(T)} .$$

By definition, add(T) is triangulated, so it follows that the stalk complex $P_3[1] \in$ add(T) and thus also $P_3[n]$ is in the triangulated category generated by add(T) for all n which proves *ii*).

Hence, T is indeed a tilting complex for A_5.

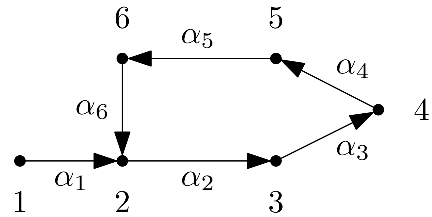
By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_5})}(T)$ is derived equivalent to A_5. And thus $A_5^{\mathrm{op}} = A_{18}$ is derived equivalent to E^{op}. We want to show that E is isomorphic to A_9.
If we use the alternating sum formula of Happel's Proposition we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$ which coincides with this one of A_9.
Now we have to define homomorphisms of complexes between the summands of T which correspond to the reversed arrows of the quiver of A_9 and show that these homomorphisms satisfy the defining relations of A_9, up to homotopy.



First we have the embedding id : $T_2 \to T_3$ (in degree zero). Moreover, we have the homomorphisms $\alpha_2\alpha_3 : T_6 \to T_2$ and $\alpha_4\alpha_5 : T_3 \to T_6$. Finally, we also have homomorphisms $\alpha_1, \alpha_4, \alpha_5, \alpha_6, \alpha_7$ and α_8 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_1\alpha_6,\ \alpha_6\alpha_7,$
$\alpha_5\alpha_6,\ \alpha_5\alpha_2\alpha_3$ and $\alpha_2\alpha_3\alpha_4\alpha_5$ are zero since they were zero in A_5. As we can see, the two paths from vertex 4 to vertex 2 and the two paths from vertex 2 to vertex 5 are the same, since we have the same commutativity relations in A_5. It is easy to see that the two paths from vertex 6 to vertex 2 are also the same. The last zero-relation $\alpha_2\alpha_3$ between vertex 6 and 3 is given by the homomorphism from T_3 to T_2 in degree zero. This is indeed a zero-relation since the homomorphism $\alpha_2\alpha_3$ is homotopic to zero.
Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_9. We have shown that they satisfy the defining relations of A_9 and that the Cartan matrices of E and A_9 coincide. From this we can conclude that $E \cong A_9$ and thus, A_9 and A_5 are derived equivalent. Since A_{17} is the opposite algebra of A_9, A_{17} is derived equivalent to $A_5^{\mathrm{op}} = A_{18}$.

3.1.2 A_{15} is derived equivalent to A_5 and A_{18}

Next consider A_{15} with the following quiver



which is derived equivalent to A_{15}^{op} since their quivers only differ at a sink/source.
Since there are arrows $1 \to 2$ and $6 \to 2$ we have homomorphisms $P_2 \xrightarrow{\alpha_1} P_1$ and $P_2 \xrightarrow{\alpha_6} P_6$. Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_{15}-modules, where $T_i : 0 \to P_i \to 0,\ i \in \{1,3,4,5,6\}$, are complexes concentrated in degree zero. Moreover, let $T_2 : 0 \to P_2 \xrightarrow{(\alpha_1,\alpha_6)} P_1 \oplus P_6 \to 0$ in degrees -1 and 0.

Now we want to show that T is a tilting complex. Since we can show like in subsection 3.1.1 that the second condition is always fulfilled for such two-term complexes we need, it suffices to prove the first one. We begin with possible maps $T_2 \to T_2[1]$ and $T_2 \to T_2[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_6)} & P_1 \oplus P_6 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_6)} & P_1 \oplus P_6 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_6)} & P_1 \oplus P_6 & \to & 0
\end{array}$$

where $\psi \in \mathrm{Hom}(P_2, P_1 \oplus P_6)$ and $(\alpha_1, 0),\ (0, \alpha_6)$ is a basis of this two-dimensional space.
The first homomorphism is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_1 \oplus P_6 \to P_2$ (as we can see in the Cartan matrix of A_{15}).

Now let $i = -1$ and consider possible maps $T_2 \to T_j[-1],\ j \neq 2$. These maps are given by a map of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_6)} & P_1 \oplus P_6 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_1, P_4, P_5, P_6 or direct sums of these. Note that there is no non-zero homomorphism $P_2 \to P_3$ since this is a zero-relation in the quiver of A_{15}.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every path from vertex $i \in \{1,4,5,6\}$ to vertex 2 ends with α_1 or α_6. Thus, every homomorphism from P_2 to P_1, P_4, P_5 or P_6 starts with α_1 or α_6, up to scalars. Hence, every homomorphism $P_2 \to Q$ can be factored through the map $(\alpha_1, \alpha_6) : P_2 \to P_1 \oplus P_6$.
Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,3,4,5,6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, let $i = 1$. We have to consider maps $T_j \to T_2[1]$ for $j \neq 2$. These are given as follows

$$\begin{array}{ccccccc}
0 & \to & Q & \to & 0 & & \\
 & & \downarrow & & & & \\
0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_6)} & P_1 \oplus P_6 & \to & 0
\end{array}$$

where Q can be either P_3, P_4, P_5 or direct sums of these.
Note that $\mathrm{Hom}(P_j, P_2) = 0$ for $j = 1$ and $j = 6$.
But no non-zero map can be zero when composed with both α_1 and α_6 since the path $\alpha_2\alpha_1$ is not a zero-relation. So the only homomorphism of complexes $T_j \to T_2[1],\ j \neq 2$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_{15}})}(T, T[i]) = 0$ in the homotopy category and that T is indeed a tilting complex for A_{15}.
Hence, T is indeed a tilting complex for A_{15}.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_{15}})}(T)$ is derived equivalent to A_{15}. And thus A_{15}^{op} is derived equivalent to E^{op}. We want to show that E is isomorphic to A_5.
Using the alternating sum formula of the Proposition by Happel we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$.
Considering the different labeling of the vertices, this is the Cartan matrix of A_5.
Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_5 (in the converse direction) and show that these homomorphisms satisfy the defining relations of A_5, up to homotopy.

$$\begin{tikzcd}
1 \arrow[r, "\alpha_1\alpha_2"] & 3 \arrow[r, "\alpha_3"] \arrow[dl, "\gamma"'] & 4 \arrow[d, "\alpha_4"] \\
2 \arrow[u, "\alpha"] \arrow[r, "\beta"'] & 6 \arrow[u, "\alpha_6\alpha_2"'] & 5 \arrow[l, "\alpha_5"]
\end{tikzcd}$$

First we have the embeddings $\alpha := (\mathrm{id}, 0) : T_1 \to T_2$ and $\beta := (0, \mathrm{id}) : T_6 \to T_2$ (in degree zero). Then we define $\gamma : T_2 \to T_3$ by the map $(0, \alpha_3\alpha_4\alpha_5) : P_1 \oplus P_6 \to P_3$ in degree 0. This is a homomorphism of complexes since $\alpha_2\alpha_3\alpha_4\alpha_5 = 0$ in A_{15}. Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_3 \to T_1$ and $\alpha_6\alpha_2 : T_3 \to T_6$. Finally, we also have homomorphisms α_3, α_4 and α_5 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_6\alpha_2\alpha_3\alpha_4$, $\alpha_4\alpha_5\alpha_6\alpha_2$ and $\alpha_5\alpha_6\alpha_2\alpha_3$ in the 4-cycle are zero since they were zero in A_{15}. As we can see, the two paths from vertex 3 to vertex 6 are the same, i.e., we here have the right commutativity relation. There is also another commutativity relation $\alpha\alpha_1\alpha_2 = \beta\alpha_6\alpha_2$ between vertex 2 and 3 which is given by the two homomorphisms from T_3 to the first and second summand of T_2. These are indeed the same paths since the homomorphism $(\alpha_2\alpha_1, 0)$ is homotopic to $(0, \alpha_2\alpha_6)$

$$\begin{tikzcd}
 & 0 \arrow[r] & P_3 \arrow[r] \arrow[dl, "\alpha_2"'] \arrow[d, shift right, "\alpha_1\alpha_2"'] \arrow[d, shift left, "\alpha_6\alpha_2"] & 0 \\
0 \arrow[r] & P_2 \arrow[r, "(\alpha_1, \alpha_6)"'] & P_1 \oplus P_6 \arrow[r] & 0
\end{tikzcd}$$

Because $\alpha_2\alpha_3\alpha_4\alpha_5 = 0$ the paths from vertex 6 to vertex 2 and from vertex 1 to 2 are zero in E. The last zero-relation is given by the concatenation of α and γ.
Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_5. We have shown that they satisfy the defining relations of A_5 and that the Cartan matrices of E and A_5 coincide. From this we can conclude that $E \cong A_5$ and thus, A_{15} and A_5 are derived equivalent. Since A_{18} is the opposite algebra of A_5, A_{18} is derived equivalent to A_{15}^{op} and since A_{15} is derived equivalent to A_{15}^{op} we get derived equivalences between A_5, A_{15} and A_{18}. With the above result, we have derived equivalences between A_5, A_9, A_{15}, A_{17} and A_{18}.

3.1.3 A_{13} is derived equivalent to A_5

The following quiver corresponds to A_{13}

$$\begin{tikzcd}
1 \arrow[r, "\alpha_1"] & 2 \arrow[d, "\alpha_2"] & 3 \arrow[l, "\alpha_7"'] \\
4 \arrow[u, "\alpha_4"] & 5 \arrow[l, "\alpha_3"] \arrow[r, "\alpha_5"'] & 6 \arrow[u, "\alpha_6"']
\end{tikzcd}$$

Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_5-modules, where $T_i : 0 \to P_i \to 0,\ i \in \{1,2,3,5,6\}$, are complexes concentrated in degree zero. Moreover, let $T_4 : 0 \to P_4 \xrightarrow{\alpha_3} P_5 \to 0$ in degrees -1 and 0.

Now we want to show that T is a tilting complex. We begin with possible maps $T_4 \to T_4[1]$ and $T_4 \to T_4[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0 \\
 & & & & \downarrow \alpha_3 & & & & \\
0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0
\end{array}$$

Here α_3 is a basis of the space of homomorphisms between P_4 and P_5.
The first homomorphism is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_5 \to P_4$ (as we can see in Cartan matrix of A_{13}).

Now let $i = -1$ and consider possible maps $T_4 \to T_j[-1],\ j \neq 4$. These maps are given by a map of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_2, P_3, P_5 or direct sums of these.
Note that there is no non-zero homomorphism $P_4 \to P_1$ and $P_4 \to P_6$ since these are zero-relation in the quiver of A_{13}.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_4 to P_2, P_3 or P_5 starts with α_3, up to scalars. Thus, every homomorphism $P_4 \to Q$ can be factored through the map $\alpha_3 : P_4 \to P_5$.
Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,2,3,5,6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, let $i = 1$. We have to consider maps $T_j \to T_4[1]$ for $j \neq 4$. These are given as follows

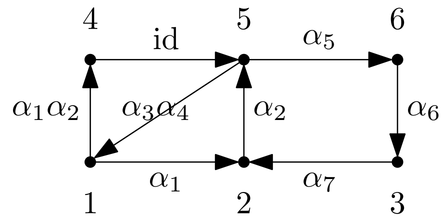
$$\begin{array}{ccccccc}
0 & \to & Q & \to & 0 & & \\
 & & \downarrow & & & & \\
0 & \to & P_4 & \xrightarrow{\alpha_3} & P_5 & \to & 0
\end{array}$$

where Q can be either P_1, P_2 or direct sums of these since $\mathrm{Hom}(P_j, P_4) = 0$ for $j = 3,\ 5$ and $j = 6$.
But no non-zero map can be zero when composed with α_3 since the path $\alpha_1\alpha_4\alpha_3 = \alpha_7\alpha_6\alpha_5 \neq 0$. So the only homomorphism of complexes $T_j \to T_4[1],\ j \neq 4$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_{13}})}(T, T[i]) = 0$ in the homotopy category.
Hence, T is indeed a tilting complex for A_{13}.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_{13}})}(T)$ is derived equivalent to A_{13}. And thus $A_{13}^{\mathrm{op}} = A_{13}$ is derived equivalent to E^{op}.
We claim that E is isomorphic to A_5 and we use the alternating sum formula of the Proposition by Happel for computing the Cartan matrix of E which is given as follows $\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$.
Considering the different labeling of the vertices, this is the Cartan matrix of A_5.

Now we define homomorphisms of complexes between the summands of T which correspond to the reversed arrows of the quiver of A_5.



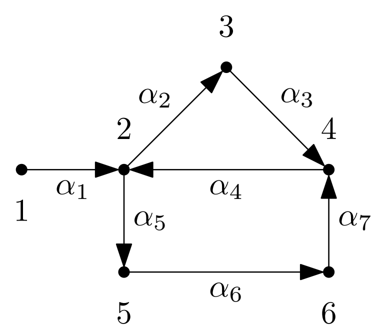
First we have the embedding id : $T_5 \to T_4$ (in degree zero). Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_4 \to T_1$ and $\alpha_3\alpha_4 : T_1 \to T_5$. Finally, we also have homomorphisms $\alpha_1, \alpha_2, \alpha_5, \alpha_6$ and α_7 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to check the relations, up to homotopy.
Clearly, the homomorphisms $\alpha_6\alpha_7\alpha_2$, $\alpha_7\alpha_2\alpha_5$, $\alpha_2\alpha_5\alpha_6$, $\alpha_2\alpha_3\alpha_4, \alpha_3\alpha_4\alpha_1$ and thus $\alpha_3\alpha_4\alpha_1\alpha_2$ are zero since they were zero in A_{13}. As we can see, the two paths $\alpha_5\alpha_6\alpha_7$ and $\alpha_3\alpha_4\alpha_1$ from vertex 5 to vertex 2 are the same since we have the same commutativity relation in A_{13}. It is easy to see, that the two path from vertex 1 to vertex 5 are also the same. The last zero-relation $\alpha_3\alpha_4$ between vertex 4 and 1 is given by the homomorphism from T_1 to T_4 in degree zero. This is indeed a zero-relation since the homomorphism $\alpha_3\alpha_4$ is homotopic to zero.
Thus, we have shown that the defined homomorphisms between the summands of T correspond to the reversed arrows of the quiver of A_5. From this we can conclude that $E \cong A_5$ and thus, A_{13} and A_5 are derived equivalent. Hence, we get derived equivalences between A_5, A_9, A_{13}, A_{15}, A_{17} and A_{18}.

Moreover, we have shown that all cluster-tilted algebras with the polynomial $4(x^6 + x^4 + x^2 + 1)$ associated to their Cartan matrix are derived equivalent.

3.2 Derived equivalences for determinant 3

3.2.1 A_3 and A_{10} are derived equivalent to A_{20}

First consider A_3 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_3-modules, where $T_i : 0 \to P_i \to 0$, $i \in \{1, 2, 3, 4, 6\}$, are complexes concentrated in degree zero and $T_5 : 0 \to P_5 \xrightarrow{\alpha_5} P_2 \to 0$ is a complex concentrated in degrees -1 and 0.

Now we want to show that T is a tilting complex.
Since condition $i)$ is obvious for all $|i| \geq 2$ we begin with possible maps $T_5 \to T_5[1]$ and $T_5 \to T_5[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\
 & & & & \downarrow \alpha_5 & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0
\end{array}$$

where α_5 is a basis of the space of homomorphisms between P_5 and P_2.
The homomorphism α_5 is homotopic to zero and in the second case there is no non-zero homomorphism $P_2 \to P_5$ (as we can see in the Cartan matrix of A_3).

Now let $i = -1$ and consider possible maps $T_5 \to T_j[-1],\ j \neq 5$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

where Q could be either P_1, P_2, P_4 or direct sums of these.
Note that there are no non-zero homomorphisms $P_5 \to P_3$ and $P_5 \to P_6$ since these are zero-relations in the quiver of A_3.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_5 to P_1, P_2 or P_4 starts with a scalar multiple of α_5. Thus, every homomorphism $P_5 \to Q$ can be factored through the map $\alpha_5 : P_5 \to P_2$.
Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 3, 4, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

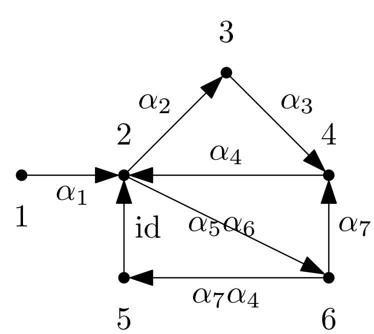
Finally, let $i = 1$. We have to consider maps $T_j \to T_5[1]$ for $j \neq 5$. These are given as follows

$$\begin{array}{ccccccc} 0 & \to & Q & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \end{array}$$

where Q can be either P_4, P_6 or direct sums of these since $\mathrm{Hom}(P_j, P_5) = 0$ for $j = 1, 2$ and $j = 3$. But no non-zero map can be zero when composed with α_5 since the path $\alpha_7\alpha_6\alpha_5 = \alpha_3\alpha_2 \neq 0$. So the only homomorphism of complexes $T_j \to T_5[1],\ j \neq 5$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_3})}(T, T[i]) = 0$ in the homotopy category.
Hence, T is indeed a tilting complex for A_3.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_3})}(T)$ is derived equivalent to A_3. And thus $A_3^{\mathrm{op}} = A_{10}$ is derived equivalent to E^{op}. We want to show that E is isomorphic to A_{20}.
Using the alternating sum formula of the Proposition by Happel we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ = 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$.
Considering the different labeling of the vertices, this is the Cartan matrix of A_{20}.
Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_{20} (in the other direction) and show that these homomorphisms satisfy the defining relations of A_{20}, up to homotopy.



First we have the embedding id : $T_2 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_5\alpha_6 : T_6 \to T_2$ and $\alpha_7\alpha_4 : T_5 \to T_6$. Finally, we also have homomorphisms $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_7 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to check the relations, up to homotopy.

Clearly, the homomorphisms $\alpha_3\alpha_4,\ \alpha_4\alpha_2,\ \alpha_4\alpha_5\alpha_6$ and $\alpha_5\alpha_6\alpha_7\alpha_4$ are zero since they were zero in A_3. As we can see, the two paths from vertex 6 to vertex 2 are the same, i.e., we here have the right commutativity relation. There is also another commutativity relation $\alpha_2\alpha_3 = \alpha_5\alpha_6\alpha_7$ between vertex 2 and 4 since these are the same paths in A_3. The concatenation of id and $\alpha_5\alpha_6$ yields to a zero-relation since the homomorphism $\alpha_5\alpha_6$ is homotopic to zero.
Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_{20}. We have shown that they satisfy the defining relations of A_{20} and that the Cartan matrices of E and A_{20} coincide. From this we can conclude that $E \cong A_{20}$ and thus, A_3 and A_{20} are derived equivalent. Since A_{20} is sink/source-equivalent to its opposite algebra, A_{20} is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_{10} and A_{20}.

3.2.2 A_3 is derived equivalent to A_{14}

Now we define a second bounded complex for A_3 by adding another two-term complex.
Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex with $T_i : 0 \to P_i \to 0,\ i \in \{1,2,3,6\}$, concentrated in degree zero and $T_4 : 0 \to P_4 \xrightarrow{(\alpha_3,\alpha_7)} P_3 \oplus P_6 \to 0$ and $T_5 : 0 \to P_5 \xrightarrow{\alpha_5} P_2 \to 0$ in degrees -1 and 0.

To show that T is a tilting complex we begin with possible maps $T_4 \to T_4[1]$ and $T_4 \to T_4[-1]$ since we've shown this for T_5 above.

$$\begin{array}{ccccccccc} & & 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 \\ & & & & \downarrow \psi & & & & \\ 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 & & \\ & & & & \downarrow 0 & & & & \\ & & 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 \end{array}$$

where $\psi \in \mathrm{Hom}(P_4, P_3 \oplus P_6)$ and $(\alpha_3, 0),\ (0, \alpha_7)$ is a basis of this two-dimensional space.
The first homomorphism is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_3 \oplus P_6 \to P_4$ (as we can see in the Cartan matrix of A_3).

Next we have a look at possible maps $T_4 \to T_5[1],\ T_4 \to T_5[-1],\ T_5 \to T_4[1]$ and $T_5 \to T_4[-1]$

$$\begin{array}{ccccccccc} & & 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 \\ & & & & \downarrow g & & & & \\ 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 & & \\ & & & & \downarrow \alpha_4 & & & & \\ & & 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 \end{array}$$

$$\begin{array}{ccccccccc} & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\ & & & & \downarrow 0 & & & & \\ 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 & & \\ & & & & \downarrow h & & & & \\ & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \end{array}$$

where g can be seen as $\alpha_2\alpha_3 = \alpha_5\alpha_6\alpha_7$ since this is a basis of the space of homomorphisms between P_4 and P_2 and h can be seen as $(0, \alpha_6)$ since this is a basis of the space of homomorphisms between $P_3 \oplus P_6$ and P_5. Moreover, α_4 is a basis of the space of homomorphisms between P_2 and P_4. As we can see, g is homotopic to zero and α_4 is not a homomorphism of complexes since $(\alpha_3\alpha_4, \alpha_7\alpha_4) = (0, \alpha_7\alpha_4) \neq 0$. With the same argument h is not a homomorphism of complexes between T_4 and $T_5[1]$. Furthermore, there is no non-zero homomorphism between P_5 and $P_3 \oplus P_6$, as we can see in the Cartan matrix of A_3.

Because we've already determined maps between T_5 and $T_j[i],\ j \notin \{4,5\}$, we consider possible maps $T_4 \to T_j[-1]$ and $T_j \to T_4[1],\ j \notin \{4,5\}$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

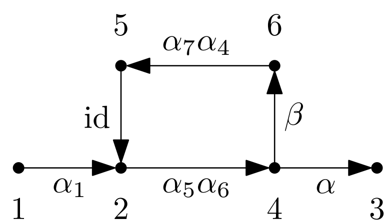
where Q could be either P_1, P_2, P_3, P_6 or direct sums of these and

$$\begin{array}{ccccccc} 0 & \to & R & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_4 & \xrightarrow{(\alpha_3,\alpha_7)} & P_3 \oplus P_6 & \to & 0 \end{array}$$

where R can be P_2 since $\mathrm{Hom}(P_j, P_4) = 0$ for $j = 1, 3$ and $j = 6$.
In the first case, there exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_4 to P_1, P_2, P_3 or P_6 starts with a scalar multiple of α_3 or α_7. Thus, every homomorphism $P_4 \to Q$ can be factored through the map $(\alpha_3, \alpha_7) : P_4 \to P_3 \oplus P_6$. In the second case, the only homomorphism of complexes $T_2 \to T_4[1]$ is the zero map since $\alpha_7\alpha_4 \neq 0$.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_3})}(T, T[i]) = 0$ in the homotopy category and that T is another tilting complex for A_3.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_3})}(T)$ is derived equivalent to A_3. And thus $A_3^{\mathrm{op}} = A_{10}$ is derived equivalent to E^{op}. We claim that E is isomorphic to A_{14}.
Using the alternating sum formula of the Proposition by Happel we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$ which coincides with the Cartan matrix of A_{14}.
Now we define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A_{14} (in the other direction).



First we have the embeddings $\mathrm{id} : T_2 \to T_5,\ \alpha := (\mathrm{id}, 0) : T_3 \to T_4$ and $\beta := (0, \mathrm{id}) : T_6 \to T_4$ (in degree zero). Moreover, we have the homomorphisms $\alpha_5\alpha_6 : T_4 \to T_2$ and $\alpha_7\alpha_4 : T_5 \to T_6$. Finally, we also have the homomorphism α_1 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to show that these homomorphisms satisfy the defining relations of A_{14}, up to homotopy. Clearly, the homomorphisms $\alpha_7\alpha_4\alpha_5\alpha_6$ and $(0, \alpha_5\alpha_6\alpha_7\alpha_4)$ in the $4-$cycle are zero since they were zero in A_3. The concatenation of $\beta,\ \alpha_7\alpha_4$ and id yields to a zero-relation since the homomorphism $(0, \alpha_7\alpha_4)$ is homotopic to zero. In the same way the concatenation of id, $\alpha_5\alpha_6$ and β yields to a zero-relation.
Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_{14}. From this we can conclude that $E \cong A_{14}$ and thus, A_3 and A_{14} are derived equivalent. Since A_{14} is its own opposite algebra, A_{14} is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_{10}, A_{14} and A_{20}.

3.2.3 A_3 is derived equivalent to A_4

The third bounded complex for A_3 is given by $T = \bigoplus_{i=1}^{6} T_i$ with $T_i : 0 \to P_i \to 0,\ i \in \{1, 3, 4, 5, 6\}$ (in degree zero) and $T_2 : 0 \to P_2 \xrightarrow{(\alpha_1,\alpha_4)} P_1 \oplus P_4 \to 0$ in degrees -1 and 0.

For showing that T is a tilting complex, we begin with possible maps $T_2 \to T_2[1]$ and $T_2 \to T_2[-1]$,

$$\begin{array}{ccccccccc} & & 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 \\ & & & & \downarrow \psi & & & & \\ 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 & & \\ & & & & \downarrow \varphi & & & & \\ & & 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 \end{array}$$

Here $\psi \in \operatorname{Hom}(P_2, P_1 \oplus P_4)$ and $(\alpha_1, 0),\ (0, \alpha_4)$ is a basis of this two-dimensional space.
But then ψ is homotopic to zero (as we can easily see). In the second case $(0, \alpha_2\alpha_3) = (0, \alpha_5\alpha_6\alpha_7)$ is a basis of the space of homomorphisms between $P_1 \oplus P_4$ and P_2. Hence, φ is not a homomorphism of complexes since $\alpha_1\alpha_2\alpha_3 = \alpha_1\alpha_5\alpha_6\alpha_7 \neq 0$.

Now let $i = -1$ and consider possible maps $T_2 \to T_j[-1],\ j \neq 2$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc} 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 \\ & & \downarrow & & & & \\ 0 & \to & Q & \to & 0 & & \end{array}$$

where Q could be either P_1, P_4, P_6 or direct sums of these.
Note that there are no non-zero homomorphisms $P_2 \to P_3$ and $P_2 \to P_5$ since these are zero-relations in the quiver of A_3.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_2 to P_1, P_4 or P_6 starts with a scalar multiple of α_1 or α_4. Thus, every homomorphism $P_2 \to Q$ can be factored through the map $(\alpha_1, \alpha_4) : P_2 \to P_1 \oplus P_4$.
Hence, $\operatorname{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 3, 4, 5, 6\}$ and thus $\operatorname{Hom}(T, T[-1]) = 0$.

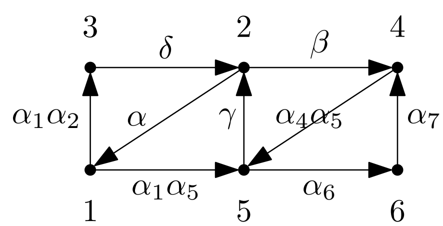
Finally, let $i = 1$. We have to consider maps $T_j \to T_2[1]$ for $j \neq 2$. These are given as follows

$$\begin{array}{ccccccc} 0 & \to & Q & \to & 0 & & \\ & & \downarrow & & & & \\ 0 & \to & P_2 & \xrightarrow{(\alpha_1,\alpha_4)} & P_1 \oplus P_4 & \to & 0 \end{array}$$

where Q can be either P_3, P_4, P_5, P_6 or direct sums of these since $\operatorname{Hom}(P_1, P_2) = 0$.
But no non-zero map can be zero when composed with both α_1 and α_4 since the paths $\alpha_2\alpha_1$ and $\alpha_5\alpha_1$ are not zero. So the only homomorphism of complexes $T_j \to T_2[1],\ j \neq 2$, is the zero map.

It follows that $\operatorname{Hom}_{\mathrm{D}^b(P_{A_3})}(T, T[i]) = 0$ in the homotopy category.
Hence, T is indeed a tilting complex for A_3.

By Rickard's theorem, $E := \operatorname{End}_{\mathrm{D}^b(P_{A_3})}(T)$ is derived equivalent to A_3. And thus $A_3^{\mathrm{op}} = A_{10}$ is derived equivalent to E^{op}. We want to show that E is isomorphic to A_4 and use the alternating sum formula of Happel's Proposition for computing the Cartan matrix of E. This Cartan matrix is given as follows $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$ and it coincides with this one of A_4.
Then the quiver of E is of the following form

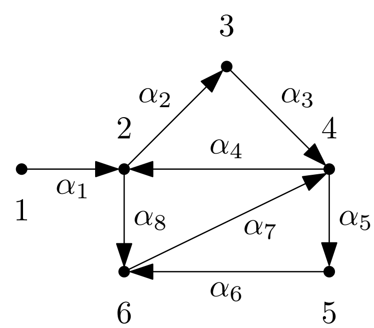


where $\alpha := (\mathrm{id}, 0) : T_1 \to T_2$ and $\beta := (0, \mathrm{id}) : T_4 \to T_2$ are the embeddings, $\gamma : T_2 \to T_5$ is defined by the map $(0, \alpha_6\alpha_7) : P_1 \oplus P_4 \to P_5$ and $\delta : T_2 \to T_3$ is defined by $(0, \alpha_3) : P_1 \oplus P_4 \to P_3$ (in degree 0). These are a homomorphisms of complexes since $\alpha_6\alpha_7\alpha_4 = 0$ and $\alpha_3\alpha_4 = 0$ in A_3. Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_3 \to T_1,\ \alpha_1\alpha_5 : T_5 \to T_1$ and $\alpha_4\alpha_5 : T_5 \to T_4$. Finally, we also have homomorphisms α_6 and α_7 as before. Note that the homomorphisms correspond to the reversed arrows. Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_4\alpha_5\alpha_6$, $\alpha_7\alpha_4\alpha_5$ and $(0, \alpha_4\alpha_5\alpha_6\alpha_7)$ are zero since they were zero in A_3. As we can see, the two paths from vertex 5 to vertex 4 are the same, i.e., we here have the right commutativity relation. There are also two other commutativity relations left. First $(0, \alpha_1\alpha_2\alpha_3) = (0, \alpha_1\alpha_5\alpha_6\alpha_7)$ between vertex 1 and 2 is one of them since these are

the same paths in A_3. Secondly, the two paths from vertex 2 to vertex 5 are the same since $(\alpha_1\alpha_5, 0)$ is homotopic to $(0, \alpha_4\alpha_5)$. It is easy to see that the concatenation of γ and α and the concatenation of δ and α are zero-relations. Finally, the path from vertex 2 to vertex 3 is zero since $(\alpha_1\alpha_2, 0)$ is homotopic to zero.
Thus, we can conclude that $E \cong A_4$ and thus, A_3 and A_4 are derived equivalent. Since $A_4 = A_4^{\mathrm{op}}$, A_4 is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_4, A_{10}, A_{14} and A_{20}.

3.2.4 A_6 is derived equivalent to A_3

First consider A_6 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_6-modules, where $T_i : 0 \to P_i \to 0,\ i \in \{1, 2, 3, 4, 6\}$, are complexes concentrated in degree zero. Moreover, let $T_5 : 0 \to P_5 \xrightarrow{\alpha_5} P_4 \to 0$ in degrees -1 and 0.

For showing that T is a tilting complex we begin with possible maps $T_5 \to T_5[1]$ and $T_5 \to T_5[-1]$,

$$\begin{array}{ccccccc}
 & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_4 & \to & 0 \\
 & & & & \downarrow \alpha_5 & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_4 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_5 & \xrightarrow{\alpha_5} & P_4 & \to & 0
\end{array}$$

Here α_5 is a basis of the space of homomorphisms between P_5 and P_4.
Then α_5 is homotopic to zero (as we can easily see). In the second case there is no non-zero homomorphism $P_4 \to P_5$.

Now let $i = -1$ and consider possible maps $T_5 \to T_j[-1],\ j \neq 5$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_3, P_4 or direct sums of these.
Note that there are no non-zero homomorphisms $P_5 \to P_1$, $P_5 \to P_2$ and $P_5 \to P_6$ since these are zero-relations in the quiver of A_6.
There exist non-zero homomorphisms of complexes between P_5 and P_3 or P_4. But they are all homotopic to zero since every homomorphism starts with a scalar multiple of α_5. Thus, every homomorphism $P_5 \to Q$ can be factored through the map $\alpha_5 : P_5 \to P_4$.
We see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 3, 4, 6\}$ and thus we have shown that
$\mathrm{Hom}(T, T[-1]) = 0$.

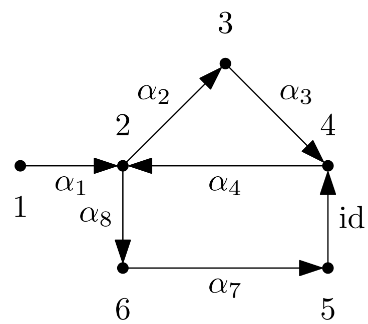
Finally, let $i = 1$. We have to consider maps $T_j \to T_5[1]$ for $j \neq 5$. These are given as follows

$$\begin{array}{ccccccc}
0 & \to & P_6 & \to & 0 & & \\
 & & \downarrow \alpha_6 & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0
\end{array}$$

since $\mathrm{Hom}(P_j, P_5) = 0$ for $j = 1, 2, 3$ and $j = 4$.
But the composition $\alpha_5\alpha_6 \neq 0$. So the only homomorphism of complexes $T_j \to T_5[1],\ j \neq 5$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_6})}(T, T[i]) = 0$ in the homotopy category and that T is indeed a tilting complex for A_6.

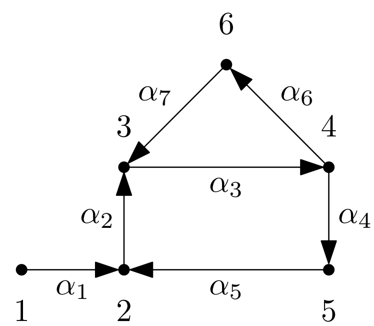
By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_6})}(T)$ is derived equivalent to A_6. And thus A_6^{op} is derived equivalent to E^{op}. We want to show that E is isomorphic to A_3.
Using the alternating sum formula of the Proposition by Happel we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$ which coincides with the Cartan matrix of A_3.
Now we have to define homomorphisms of complexes between the summands of T which correspond to the reversed arrows of the quiver of A_3.



First we have the embedding $\mathrm{id} : T_4 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_7$ and α_8 as before. Since all the relations are the same as in A_6 we have shown that they satisfy the defining relations of A_3. From this we can conclude that $E \cong A_3$ and thus, A_3 and A_6 are derived equivalent. Since A_6^{op} is sink/source-equivalent to A_{21}, A_{21} is also derived equivalent to $A_3^{\mathrm{op}} = A_{10}$. Hence, we get derived equivalences between A_3, A_4, A_6, A_{10}, A_{14}, A_{20} and A_{21}.

3.2.5 A_{16} is derived equivalent to A_6

Now consider A_{16} with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_{16}-modules, where $T_i : 0 \to P_i \to 0,\ i \in \{1, 2, 3, 4, 6\}$, are complexes concentrated in degree zero and $T_5 : 0 \to P_5 \xrightarrow{\alpha_4} P_4 \to 0$ is a complex concentrated in degrees -1 and 0.

Now we want to show that T is a tilting complex and we begin with possible maps $T_5 \to T_5[1]$ and $T_5 \to T_5[-1]$,

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 \\
 & & & & \downarrow \alpha_4 & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0
\end{array}$$

Here α_4 is a basis of the space of homomorphisms between P_5 and P_4.
The homomorphism α_4 is homotopic to zero and in the second case there is no non-zero homomorphism $P_4 \to P_5$ (as we can see in the Cartan matrix of A_{16}).

Now let $i = -1$ and consider possible maps $T_5 \to T_j[-1],\ j \neq 5$. These are given by maps of complexes as follows

$$\begin{array}{ccccccc}
0 & \to & P_5 & \xrightarrow{\alpha_4} & P_4 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_3, P_4 or direct sums of these.
Note that there are no non-zero homomorphisms $P_5 \to P_1,\ P_5 \to P_2$ and $P_5 \to P_6$ since these are zero-relations in the quiver of A_{16}.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_5 to P_3 or P_4 starts with a scalar multiple of α_4. Thus, every homomorphism $P_5 \to Q$ can be factored through the map $\alpha_4 : P_5 \to P_4$.
Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 3, 4, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

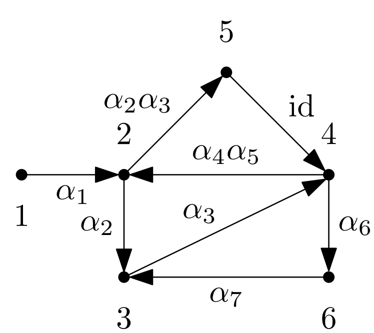
Finally, let $i = 1$. We have to consider maps $T_j \to T_5[1]$ for $j \neq 5$. These are given as follows

$$\begin{array}{ccccccc}
0 & \to & Q & \to & 0 & & \\
 & & \downarrow & & & & \\
0 & \to & P_5 & \xrightarrow{\alpha_5} & P_2 & \to & 0
\end{array}$$

where Q can be either P_2, P_3 or direct sums of these since $\mathrm{Hom}(P_j, P_5) = 0$ for $j = 1, 4$ and $j = 6$. But no non-zero map can be zero when composed with α_4 since the path $\alpha_2\alpha_5\alpha_4 = \alpha_7\alpha_6 \neq 0$. So the only homomorphism of complexes $T_j \to T_5[1],\ j \neq 5$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_{16}})}(T, T[i]) = 0$ and that T is indeed a tilting complex for A_{16}.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_{16}})}(T)$ is derived equivalent to A_{16}. And thus A_{16}^{op} is derived equivalent to E^{op}. Since we want to show that E is isomorphic to A_6, we use the alternating sum formula of Happel's Proposition and compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$.
Considering the different labeling of the vertices, this is the Cartan matrix of A_6.
Then the quiver of E is of the following form



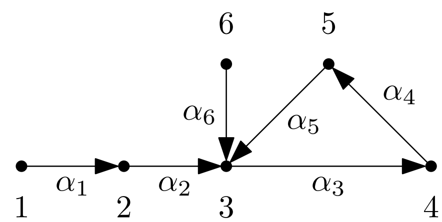
where we have the embedding id : $T_4 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_2\alpha_3 : T_5 \to T_2$ and $\alpha_4\alpha_5 : T_2 \to T_4$. Finally, we also have homomorphisms $\alpha_1, \alpha_2, \alpha_3, \alpha_6$ and α_7 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $\alpha_7\alpha_3,\ \alpha_3\alpha_6,$
$\alpha_3\alpha_4\alpha_5$ and $\alpha_4\alpha_5\alpha_2\alpha_3$ are zero since they were zero in A_{16}. As we can see, the two paths from vertex 2 to vertex 4 are the same, i.e., we here have the right commutativity relation. There is also another commutativity relation $\alpha_6\alpha_7 = \alpha_4\alpha_5\alpha_2$ between vertex 4 and 3 since these are the same paths in A_{16}. The path from vertex 5 to vertex 2 is the last zero-relation since the homomorphism $\alpha_4\alpha_5$ is homotopic to zero.
Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A_6. From this we can conclude that $E \cong A_6$ and thus, A_6 and A_{16} are derived equivalent. Since A_{16}^{op} is sink/source-equivalent to A_{19}, A_{19} is also derived equivalent to $A_6^{\mathrm{op}} \underset{\mathrm{der}}{\sim} A_{21}$.

Hence, we get derived equivalences between all cluster-tilted algebras with determinant 3.

3.3 Derived equivalences for polynomial $2(x^6 - x^4 + 2x^3 - x^2 + 1)$

3.3.1 A_7 is derived equivalent to A_2

Let A_7' be the following cluster-tilted algebra which is sink/source equivalent to A_7 (at the vertex 6).



Let $T = \bigoplus_{i=1}^{6} T_i$ be the following bounded complex of projective A_7'-modules.
Let $T_i : 0 \to P_i \to 0,\ i \in \{1, 2, 4, 5, 6\}$, be complexes concentrated in degree zero and $T_3 : 0 \to P_3 \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} P_2 \oplus P_5 \oplus P_6 \to 0$ in degrees -1 and 0.

Now we want to show that T is a tilting complex and we begin with possible maps $T_3 \to T_3[1]$ and $T_3 \to T_3[-1]$,

$$
\begin{array}{ccccccccc}
 & & 0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 & & \\
 & & & & \downarrow 0 & & & & \\
 & & 0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0
\end{array}
$$

where $\psi \in \mathrm{Hom}(P_3, P_2 \oplus P_5 \oplus P_6)$ and $(\alpha_2, 0, 0),\ (0, \alpha_5, 0),\ (0, 0, \alpha_6)$ is a basis of this three-dimensional space of homomorphisms.
Then homomorphism ψ is homotopic to zero and in the second case there is no non-zero homomorphism $P_2 \oplus P_5 \oplus P_6 \to P_3$.

Now let $i = -1$ and consider possible maps $T_3 \to T_j[-1],\ j \neq 3$. These maps are given by a map of complexes as follows

$$
\begin{array}{ccccccc}
0 & \to & P_3 & \overset{(\alpha_2,\alpha_5,\alpha_6)}{\longrightarrow} & P_2 \oplus P_5 \oplus P_6 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}
$$

where Q could be either P_1, P_2, P_5, P_6 or direct sums of these.
Note that there is no non-zero homomorphism $P_3 \to P_4$ since this is a zero-relation in the quiver of A_7.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_3 to P_1, P_2, P_5 or P_6 starts with $\alpha_2,\ \alpha_5$ or α_6, up to scalars. Thus, every homomorphism

$P_3 \to Q$ can be factored through the map $(\alpha_2, \alpha_5, \alpha_6) : P_3 \to P_2 \oplus P_5 \oplus P_6$.
Directly from the definition we see that $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1, 2, 4, 5, 6\}$ and thus we have shown that $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, let $i = 1$. We have to consider maps $T_j \to T_3[1]$ for $j \neq 3$. But these are given as follows

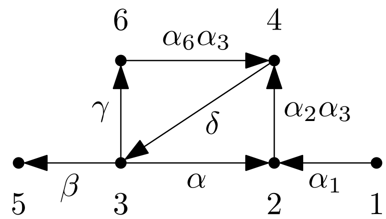
$$\begin{array}{ccccccc} 0 & \to & P_4 & \to & 0 & & \\ & & \downarrow \alpha_3 & & & & \\ 0 & \to & P_3 & \xrightarrow{(\alpha_2,\alpha_5,\alpha_6)} & P_2 \oplus P_5 \oplus P_6 & \to & 0 \end{array}$$

since $\mathrm{Hom}(P_j, P_3) = 0$ for $j = 1, 2, 5$ and $j = 6$.
But the concatenation of $(\alpha_2, \alpha_5, \alpha_6)$ and α_3 is not zero since $\alpha_2\alpha_3 \neq 0$ and $\alpha_6\alpha_3 \neq 0$. So the only homomorphism of complexes $T_j \to T_3[1],\ j \neq 3$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A'_7})}(T, T[i]) = 0$ in the homotopy category and that T is indeed a tilting complex for A'_7.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A'_7})}(T)$ is derived equivalent to A_7. We want to show that E is isomorphic to the algebra A'_2 obtained from A_2 by sink/source equivalences at the vertices 1 and 4. Using the alternating sum formula of the Proposition by Happel we can compute the Cartan matrix of E to be $\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$ which coincides with the Cartan matrix of A'_2 (up to permutation).

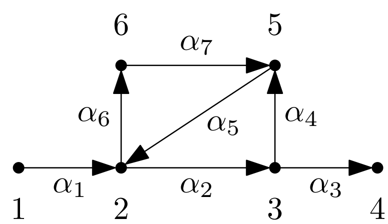
Now we define homomorphisms of complexes between the summands of T which correspond to the reversed arrows of the quiver of A'_2.



First we have the embeddings $\alpha := (\mathrm{id}, 0, 0) : T_2 \to T_3$, $\beta := (0, \mathrm{id}, 0) : T_5 \to T_3$ and $\gamma := (0, 0, \mathrm{id}) : T_6 \to T_3$ (in degree zero). Then we define $\delta : T_3 \to T_4$ by the map $(0, \alpha_4, 0) : P_2 \oplus P_5 \oplus P_6 \to P_4$ in degree 0. This is a homomorphism of complexes since $\alpha_4\alpha_5 = 0$ in A_7. Moreover, we have the homomorphisms $\alpha_2\alpha_3 : T_4 \to T_2$ and $\alpha_6\alpha_3 : T_4 \to T_6$. Finally, we also have the homomorphism α_1 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to check the relations, up to homotopy. Clearly, the homomorphisms $(0, \alpha_6\alpha_3\alpha_4, 0)$ and $(0, \alpha_2\alpha_3\alpha_4, 0)$ are zero since they were zero in A_7. As we can see, the paths from vertex 4 to vertex 2 and to vertex 6 are zero. There is one commutativity relation left. The two paths from vertex 3 to vertex 4 are the same since $(0, 0, \alpha_6\alpha_3)$ is homotopic to $(\alpha_2\alpha_3, 0, 0)$.
From this we can conclude that $E \cong A'_2$ and thus, A_7 and A_2 are derived equivalent.

3.3.2 A_2 is derived equivalent to A_{12}

Now we consider A_2 with the following quiver



Let $T = \bigoplus_{i=1}^{6} T_i$ be the complex with $T_i : 0 \to P_i \to 0,\ i \in \{1,3,4,5,6\}$, concentrated in degree zero and $T_2 : 0 \to P_2 \overset{(\alpha_1,\alpha_5)}{\longrightarrow} P_1 \oplus P_5 \to 0$ in degrees -1 and 0.

To show that T is a tilting complex we begin with possible maps $T_2 \to T_2[1]$ and $T_2 \to T_2[-1]$

$$\begin{array}{ccccccccc}
 & & 0 & \to & P_2 & \overset{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 \\
 & & & & \downarrow \psi & & & & \\
0 & \to & P_2 & \overset{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 & & \\
 & & & & \downarrow \varphi & & & & \\
 & & 0 & \to & P_2 & \overset{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0
\end{array}$$

Here $\psi \in \mathrm{Hom}(P_2, P_1 \oplus P_5)$ and $(\alpha_1, 0),\ (0, \alpha_5)$ is a basis of this two-dimensional space.
But then ψ is homotopic to zero (as we can easily see). In the second case $(0, \alpha_6\alpha_7) = (0, \alpha_2\alpha_4)$ is a basis of the space of homomorphisms between $P_1 \oplus P_5$ and P_2. Hence, φ is not a homomorphism of complexes since $\alpha_1\alpha_6\alpha_7 = \alpha_1\alpha_2\alpha_4 \neq 0$.

Now let $i = -1$ and consider possible maps $T_2 \to T_j[-1],\ j \neq 2$. These are given by maps of complexes as follows

$$\begin{array}{cccccc}
0 & \to & P_2 & \overset{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0 \\
 & & \downarrow & & & & \\
0 & \to & Q & \to & 0 & &
\end{array}$$

where Q could be either P_1, P_5 or direct sums of these.
Note that there are no non-zero homomorphisms $P_2 \to P_3,\ P_2 \to P_4$ and $P_2 \to P_6$ since these are zero-relations in the quiver of A_2.
There exist non-zero homomorphisms of complexes. But they are all homotopic to zero since every homomorphism from P_2 to P_1 or P_5 starts with a scalar multiple of α_1 or α_5. Thus, every homomorphism $P_2 \to Q$ can be factored through the map $(\alpha_1, \alpha_5) : P_2 \to P_1 \oplus P_5$.
Hence, $\mathrm{Hom}(T, T_j[-1]) = 0$ for $j \in \{1,3,4,5,6\}$ and thus $\mathrm{Hom}(T, T[-1]) = 0$.

Finally, let $i = 1$. We have to consider maps $T_j \to T_2[1]$ for $j \neq 2$. These are given as follows

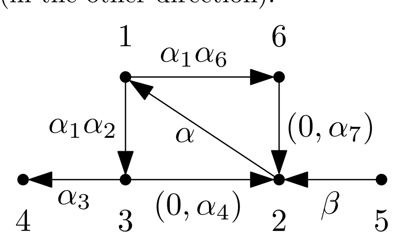
$$\begin{array}{ccccccc}
0 & \to & Q & \to & 0 & & \\
 & & \downarrow & & & & \\
0 & \to & P_2 & \overset{(\alpha_1,\alpha_5)}{\longrightarrow} & P_1 \oplus P_5 & \to & 0
\end{array}$$

where Q can be either P_3, P_4, P_5, P_6 or direct sums of these since $\mathrm{Hom}(P_1, P_2) = 0$.
But no non-zero map can be zero when composed with both α_1 and α_5 since the paths $\alpha_2\alpha_1$ and $\alpha_6\alpha_1$ are not zero. So the only homomorphism of complexes $T_j \to T_2[1],\ j \neq 2$, is the zero map.

It follows that $\mathrm{Hom}_{\mathrm{D}^b(P_{A_2})}(T, T[i]) = 0$ in the homotopy category.
Hence, T is a tilting complex for A_2.

By Rickard's theorem, $E := \mathrm{End}_{\mathrm{D}^b(P_{A_2})}(T)$ is derived equivalent to A_2. We show that E is isomorphic to A'_{12}, the algebra obtained from A_{12} via sink/source equivalence (at the vertex 6). Using the alternating sum formula of Happel's Proposition we compute the Cartan matrix of E to be $\begin{pmatrix} 1&1&1&1&1&1 \\ 1&1&0&0&1&0 \\ 0&1&1&1&1&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&1&0&0&1&1 \end{pmatrix}$ which coincides with the Cartan matrix of A'_{12} (up to permutation).

Now we have to define homomorphisms of complexes between the summands of T which correspond to the arrows of the quiver of A'_{12} (in the other direction).



First we have the embeddings $\alpha := (\mathrm{id}, 0) : T_2 \to T_1$ and $\beta := (0, \mathrm{id}) : T_2 \to T_5$ (in degree zero). Moreover, we have the homomorphisms $\alpha_1\alpha_2 : T_3 \to T_1$, $\alpha_1\alpha_6 : T_6 \to T_1$, $(0, \alpha_4) : T_2 \to T_3$ and $(0, \alpha_7) : T_2 \to T_6$. Finally, we also have the homomorphism α_3 as before. Note that the homomorphisms correspond to the reversed arrows.
Now we have to show that these homomorphisms satisfy the defining relations of A'_{12}, up to homotopy. Clearly, the concatenation of $(0, \alpha_4)$ and α and the concatenation of $(0, \alpha_7)$ and α are zero-relations. It is easy to see, that the two paths from vertex 1 to vertex 2 are the same since $\alpha_1\alpha_6\alpha_7 = \alpha_1\alpha_2\alpha_4$. The two paths from vertex 2 to vertex 3 and from vertex 2 to vertex 6 are zero since $(\alpha_1\alpha_2, 0)$ and $(\alpha_1\alpha_6, 0)$ are homotopic to zero.
Thus, we defined homomorphisms between the summands of T corresponding to the reversed arrows of the quiver of A'_{12}. From this we can conclude that $E \cong A'_{12}$ and thus, A_2 and A_{12} are derived equivalent.

Hence, we get derived equivalences between A_2, A_7 and A_{12}.

A Cluster-tilted algebras of type E_7

First we list all quivers of the cluster-tilted algebras of type E_7. Algebras with the same polynomial associated with their Cartan matrix are grouped in one table.

Note that a tuple (a, b) stands for an arrow $a \to b$ and that the numbering of the algebras in the tables results from the numbering of the whole list.

$\boldsymbol{x^7 - x^6 + x^4 - x^3 + x - 1}$	
algebra KQ/I	quiver Q
A_1	$(1,2), (2,3), (3,4), (4,5), (4,7), (5,6)$

$\boldsymbol{2(x^7 - x^5 + 2x^4 - 2x^3 + x^2 - 1)}$	
algebra KQ/I	quiver Q
A_2	$(1,2), (2,3), (3,4), (4,5), (4,7), (5,6), (6,4)$
A_{13}	$(1,2), (2,3), (3,4), (4,5), (4,7), (5,3), (5,6), (6,4)$
A_{20}	$(1,2), (2,3), (3,4), (4,5), (5,3), (5,6), (5,7), (6,4)$

$\boldsymbol{2(x^7 - x^5 + x^4 - x^3 + x^2 - 1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_3	$(2,1), (3,2), (3,4),$ $(5,3), (5,6), (6,7), (7,5)$	A_4	$(2,1), (3,2), (3,4),$ $(3,5), (5,6), (6,3), (7,6)$
A_5	$(2,1), (3,2), (3,4),$ $(3,7), (4,5), (5,3), (6,4)$	A_{12}	$(2,1), (2,3), (3,4), (4,2),$ $(4,5), (5,3), (6,4), (7,6)$
A_{16}	$(1,2), (2,5), (3,2), (3,6),$ $(4,2), (5,3), (5,4), (7,5)$	A_{25}	$(1,2), (2,3), (3,5), (4,3),$ $(5,4), (5,6), (6,3), (6,7)$

$\boldsymbol{2(x^7 - 2x^5 + 4x^4 - 4x^3 + 2x^2 - 1)}$	
algebra KQ/I	quiver Q
A_{18}	$(1,2), (2,3), (3,4), (4,5), (5,3), (5,6), (6,4), (6,7)$

$\boldsymbol{3(x^7 - 1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_6	$(1,2), (2,3), (3,4),$ $(4,5), (4,7), (5,6), (6,3)$	A_7	$(1,2), (2,3), (3,4),$ $(4,5), (5,6), (6,3), (6,7)$
A_8	$(1,2), (2,3), (3,4),$ $(3,7), (4,5), (5,2), (6,4)$	A_{17}	$(1,2), (2,3), (3,4), (3,7)$ $(4,5), (5,6), (6,3), (7,6)$
A_{19}	$(1,2), (2,3), (3,4), (4,5),$ $(5,6), (6,3), (6,7), (7,5)$	A_{21}	$(1,2), (2,3), (3,4), (3,7),$ $(4,2), (4,5), (5,6), (6,3)$
A_{23}	$(1,2), (2,3), (3,4), (4,5),$ $(5,6), (5,7), (6,3), (7,4)$	A_{26}	$(1,2), (2,3), (3,4), (4,5),$ $(4,7), (5,3), (6,4), (7,2)$
A_{27}	$(1,2), (2,3), (3,4), (4,2),$ $(4,5), (5,6), (6,3), (6,7)$	A_{28}	$(1,2), (2,4), (3,2), (4,3),$ $(4,6), (5,2), (6,5), (7,6)$
A_{29}	$(2,1), (2,3), (3,4), (4,5)$ $(5,2), (5,6), (6,4), (7,6)$	A_{36}	$(1,2), (2,3), (3,4), (4,5),$ $(4,7), (5,6), (6,3), (7,3)$
A_{37}	$(2,1), (2,3), (3,4), (3,5),$ $(4,2), (5,6), (6,2), (7,3)$	A_{39}	$(1,2), (2,3), (3,4), (4,2),$ $(4,5), (4,7), (5,6), (6,3)$
A_{44}	$(1,2), (2,3), (3,4), (3,7),$ $(4,5), (5,3), (5,6), (6,7), (7,5)$	A_{47}	$(2,1), (2,3), (3,4), (3,5)$ $(4,2), (5,2), (5,6), (6,3), (7,4)$
A_{51}	$(2,1), (2,3), (2,5), (3,4),$ $(4,2), (5,4), (5,6), (6,2), (7,6)$	A_{52}	$(1,2), (2,3), (3,4), (3,6)$ $(4,5), (5,3), (6,5), (6,7), (7,3)$
A_{54}	$(1,2), (2,3), (3,4), (4,2),$ $(4,5), (5,3), (5,6), (5,7), (6,4)$	A_{56}	$(1,2), (2,3), (3,5), (4,3),$ $(5,4), (5,6), (6,3), (6,7), (7,5)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{59}	(2,1), (2,3), (3,4), (3,5), (4,2), (5,2), (5,6), (6,3), (6,7)	A_{60}	(1,2), (2,3), (2,7), (3,1), (3,4), (4,5), (5,2), (5,6), (6,4)
A_{66}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,7), (5,3), (5,6), (6,4)	A_{67}	(1,2), (2,3), (2,6), (3,4), (4,2), (4,5), (5,3), (6,4), (7,4)
A_{72}	(2,1), (2,3), (2,6), (3,4) (3,7), (4,2), (4,5), (5,6), (6,4)	A_{73}	(2,1), (2,3), (2,7), (3,4), (4,5), (4,6), (5,3), (6,2), (7,6)
A_{75}	(1,2), (2,3), (2,5), (3,4), (4,2), (5,4), (5,6), (6,2), (7,4)	A_{86}	(1,2), (2,3), (2,7), (3,4), (3,6), (4,5), (5,3), (6,2), (6,5), (7,6)
A_{87}	(1,2), (2,3), (3,4), (3,5), (4,6) (5,2), (5,6), (6,3), (6,7), (7,4)	A_{89}	(1,2), (2,3), (3,4), (4,2), (4,5), (5,3), (5,6), (6,4), (6,7), (7,5)
A_{97}	(1,2), (2,5), (3,2), (3,6), (4,2), (5,3), (5,4), (6,5), (6,7), (7,3)		

$\mathbf{4(x^7+x^6-x^5+x^4-x^3+x^2-x-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{14}	(2,1), (2,3), (3,4), (4,2), (5,2), (5,6), (6,7), (7,5)	A_{15}	(2,1), (2,3), (2,5), (3,4), (4,2), (5,6), (6,2), (7,6)
A_{22}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,7), (5,6), (6,4)	A_{31}	(2,1), (2,3), (2,5), (3,4), (4,2), (5,6), (6,7), (7,5)
A_{46}	(2,1), (2,3), (3,4), (3,5), (4,2), (5,2), (5,6), (6,7), (7,5)	A_{57}	(2,1), (2,3), (2,5), (3,4), (4,2), (5,4), (5,6), (6,7), (7,5)

$\mathbf{4(x^7+x^6-x^5-x^4+x^3+x^2-x-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{45}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,6), (5,3), (6,7), (7,4)	A_{50}	(2,1), (2,3), (3,4), (3,6), (4,2), (4,5), (5,3), (6,7), (7,3)

$\mathbf{4(x^7+x^6-2x^5+2x^4-2x^3+2x^2-x-1)}$	
algebra KQ/I	quiver Q
A_{53}	(2,1), (2,3), (3,4), (4,2), (4,5), (5,3), (5,6), (6,7), (7,5)

$\mathbf{4(x^7+x^5-x^4+x^3-x^2-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_9	(1,2), (2,3), (3,4), (4,5), (5,6), (6,7), (7,3)	A_{10}	(1,2), (2,3), (3,4), (4,5), (5,6), (6,2), (7,4)
A_{30}	(2,1), (2,4), (3,2), (4,3), (4,5), (5,6), (6,7), (7,2)	A_{33}	(2,1), (2,3), (3,4), (4,5), (5,2), (5,6), (6,7), (7,4)
A_{34}	(1,2), (2,3), (3,4), (4,5), (4,6), (5,2), (6,7), (7,3)	A_{40}	(2,1), (2,3), (3,4), (4,5), (5,6), (5,7), (6,2), (7,4)
A_{43}	(2,1), (2,3), (2,7), (3,4), (4,5), (5,6), (6,2), (7,6)	A_{48}	(1,2), (2,3), (2,7), (3,4), (4,2), (4,5), (5,6), (6,7), (7,4)
A_{58}	(2,1), (2,3), (3,4), (3,5), (4,2), (5,6), (5,7), (6,2), (7,3)	A_{61}	(1,2), (2,3), (3,4), (4,2), (4,5), (5,3), (5,6), (6,7), (7,4)
A_{63}	(1,2), (2,3), (3,4), (4,5), (4,6), (5,2), (6,3), (6,7), (7,4)	A_{64}	(1,2), (2,3), (3,4), (4,5), (4,7), (5,2), (5,6), (6,4), (7,6)
A_{68}	(1,2), (2,3), (2,6), (2,7), (3,1), (3,4), (4,5), (5,2), (6,5)	A_{69}	(1,2), (2,3), (3,4), (3,6), (4,2), (4,5), (5,3), (6,7), (7,5)
A_{70}	(1,2), (2,3), (3,4), (3,7), (4,5), (4,6), (5,2), (6,3), (7,6)	A_{76}	(2,1), (2,3), (2,4), (3,6), (4,5), (5,6), (6,2), (6,7), (7,5)
A_{77}	(1,2), (1,4), (2,6), (3,2), (4,5), (5,1), (6,5), (6,7), (7,3)	A_{78}	(1,2), (2,5), (3,2), (3,7), (4,3), (5,6), (6.3), (7,4), (7,6)
A_{80}	(1,2), (2,6), (3,2), (3,4), (4,5), (5,6), (6,3), (6,7), (7,2)	A_{85}	(1,2), (2,3), (2,4), (3,5), (4,5), (5,2), (5,6), (6,4), (6,7), (7,5)
A_{88}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,7), (5,3), (5,6), (6,4), (7,6)	A_{91}	(1,2), (2,3), (3,4), (3,6), (4,2) (4,5), (5,3), (6,5), (6,7), (7,3)
A_{92}	(1,2), (2,5), (3,2), (3,7), (4,3), (5,1), (5,6), (6,3), (7,4), (7,6)	A_{99}	(2,1), (2,5), (3,2), (3,4), (4,5), (5,3), (5,6), (5,7), (6,4), (7,2)
A_{100}	(1,5), (2,1), (2,6), (3,2), (3,4), (4,7), (5,2), (6,5), (6,7), (7,3)	A_{101}	(1,2), (2,3), (2,5), (3,6), (4,1), (5,4), (5,6), (6,2), (6,7), (7,3)
A_{103}	(1,2), (2,6), (3,2), (3,7), (4,3), (5,1), (6,3), (6,5), (7,4), (7,6)	A_{109}	(1,2), (1,4), (2,3), (2,5), (3,6), (4,5), (5,1), (5,6), (6,2), (6,7), (7,3)
A_{110}	(1,4), (2,1), (2,3), (2,5), (3,6), (4,2), (5,4), (5,6), (6,2), (6,7), (7,3)	A_{111}	(1,2), (2,3), (2,4), (3,5), (4,1), (4,5), (5,2), (5,6), (6,3), (6,7), (7,5)

$\mathbf{4(x^7+x^5-2x^4+2x^3-x^2-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{38}	(2,1), (2,3), (3,4), (4,5), (4,7), (5,6), (6,2), (7,3)	A_{41}	(2,1), (2,3), (3,4), (4,5), (5,6), (6,2), (6,7), (7,5)
A_{71}	(2,1), (2,3), (3,4), (4,5), (4,6), (5,3), (6,2), (6,7), (7,4)	A_{95}	(1,6), (2,1), (3,2), (3,7), (4,3), (5,1), (6,3), (6,5), (7,4), (7,6)
A_{98}	(1,2), (2,3), (2,5), (3,7), (4,3), (5,1), (5,6), (6,7), (7,2), (7,4)		

$\mathbf{5(x^7+x^5-x^4+x^3-x^2-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{11}	$(2,1),(2,3),(3,4),$ $(4,5),(5,6),(6,7),(7,2)$	A_{42}	$(1,2),(2,3),(3,4),(4,1)$ $(4,5),(5,6),(6,7),(7,3)$
A_{65}	$(1,2),(1,7),(2,3),(3,1),$ $(3,4),(4,5),(5,6),(6,7),(7,3)$	A_{79}	$(1,2),(2,3),(3,4),(4,1)$ $(4,5),(5,6),(5,7),(6,3),(7,4)$
A_{81}	$(1,2),(2,3),(3,4),(3,7),$ $(4,1),(4,5),(5,3),(6,5),(7,6)$	A_{82}	$(1,2),(2,3),(3,4),(3,7),$ $(4,1),(4,5),(5,6),(6,3),(7,6)$
A_{83}	$(1,2),(2,3),(3,4),(3,7),$ $(4,5),(4,6),(5,1),(6,3),(7,6)$	A_{90}	$(1,2),(2,5),(3,2),(3,6),(4,1),$ $(4,7),(5,3),(5,4),(6,5),(7,5)$
A_{94}	$(2,1),(2,3),(2,6),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,2)$	A_{102}	$(1,5),(2,1),(2,3),(3,6),(4,3),$ $(4,7),(5,6),(6,2),(6,4),(7,6)$
A_{105}	$(1,3),(2,1),(2,4),(2,7),(3,2)$ $(4,5),(5,2),(6,5),(7,3),(7,6)$	A_{106}	$(1,2),(2,3),(3,1),(3,4),(3,5),$ $(4,7),(5,2),(5,6),(6,3),(7,6)$
A_{107}	$(1,3),(2,1),(2,6),(3,2),(3,7)$ $(4,3),(5,2),(6,3),(6,5),(7,4),(7,6)$	A_{108}	$(1,7),(2,1),(2,3),(2,6),(3,4),$ $(4,2),(4,5),(5,6),(6,4),(6,7),(7,2)$
A_{112}	$(1,2),(1,6),(2,3),(3,1),(3,4),$ $(3,5),(4,2),(5,6),(5,7),(6,3),(7,3)$		

$\mathbf{6(x^7+x^6-x^4+x^3-x-1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{24}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(6,7),(7,5)$	A_{32}	$(2,1),(2,3),(3,4),(3,6),$ $(4,5),(5,2),(6,7),(7,3)$
A_{49}	$(1,2),(2,3),(3,1),(3,4),$ $(3,6),(4,5),(5,2),(6,7),(7,3)$	A_{55}	$(1,2),(2,3),(3,1),(3,4),$ $(4,5),(5,6),(6,3),(6,7),(7,5)$
A_{62}	$(1,2),(2,3),(3,1),(3,4),$ $(4,5),(5,6),(5,7),(6,3),(7,4)$	A_{74}	$(1,2),(2,3),(2,4),(3,1),$ $(4,5),(4,6),(5,2),(6,7),(7,2)$
A_{84}	$(1,2),(2,3),(3,1),(3,4),(3,6),$ $(4,5),(5,3),(5,7),(6,5),(7,6)$	A_{93}	$(1,5),(2,1),(2,3),(3,5),(4,1),$ $(5,2),(5,4),(5,7),(6,5),(7,6)$
A_{96}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(4,6),(5,3),(6,3),(6,7),(7,4)$		

$\mathbf{6(x^7+x^5-x^2-1)}$	
algebra KQ/I	quiver Q
A_{104}	$(1,2),(2,3),(2,5),(3,6),(4,1),(5,4),(5,6),(6,2),(6,7),(7,5)$

$\mathbf{8(x^7+x^6+x^5-x^4+x^3-x^2-x-1)}$	
algebra KQ/I	quiver Q
A_{35}	$(1,2),(2,3),(3,1),(3,4),(4,5),(5,6),(6,7),(7,3)$

B Derived equivalences for cluster-tilted algebras of type E_7

First we list the opposite algebra for each cluster-tilted algebra. By a result of Rickard [18, Prop. 9.1], if A is derived equivalent to B, also A^{op} is derived equivalent to B^{op}.

After this, we list the cluster-tilted algebra, the corresponding tilting complex, the derived equivalent cluster-tilted algebra with permutation of the vertices (up to sink/source equivalence) and the resulting equivalence for the opposite algebras (if necessary).

The tilting complexes are of the following form: If we have a tilting complex $T = \bigoplus_{i=1}^{7} T_i$ with $T_i : 0 \to P_i \to 0,\ i \in \{1,3,4,5,6,7\}$ (in degree zero) and $T_2 : 0 \to P_2 \to P_1 \oplus P_5 \to 0$ in degrees -1 and 0 we write $(2;1,5)$ for T_2 and know that the other summands are just the stalk complexes.

We write the permutation as a product of disjoint cycles. If we have a permutation $(135)(67)$ the labeling of the vertices changes as follows:
$1 \to 3$, $3 \to 5$, $5 \to 1$, $6 \to 7$, $7 \to 6$ and the labeling of the other vertices is left unchanged.

B.1 Polynomial $2(x^7 - x^5 + 2x^4 - 2x^3 + x^2 - 1)$

$A_2^{\text{op}} \underset{\text{s/s}}{\sim} A_2$, $A_{13}^{\text{op}} \underset{\text{s/s}}{\sim} A_{20}$

$A_{13}(*)$	$(4;3,6,7)$	$\underset{\text{der}}{\sim}$	A_2	(567)	$\Rightarrow$	$A_{20} \underset{\text{der}}{\sim} A_2$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.2 Polynomial $2(x^7 - x^5 + x^4 - x^3 + x^2 - 1)$

$A_3^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_3$, $A_4^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_5$, $A_{12}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{25}$, $A_{16}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{16}$

A_{16}	$(2;1,3,4)$	$\underset{\mathrm{der}}{\sim}$	A_4	$(156)(23)$	$\Rightarrow$	$A_{16} \underset{\mathrm{der}}{\sim} A_5$
A_{25}	$(3;2,4,6)$	$\underset{\mathrm{der}}{\sim}$	A_5	(16247)	$\Rightarrow$	$A_{12} \underset{\mathrm{der}}{\sim} A_4$
A_5	$(4;3,6)$	$\underset{\mathrm{der}}{\sim}$	A_3	(457)	$\Rightarrow$	$A_4 \underset{\mathrm{der}}{\sim} A_3$

B.3 Polynomial $3(x^7 - 1)$

$A_6^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_7$, $A_8^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_8$, $A_{17}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{36}$, $A_{19}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{23}$, $A_{21}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{39}$, $A_{26}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{28}$, $A_{27}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{29}$, $A_{37}^{\mathrm{op}} = A_{37}$, $A_{44}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{56}$, $A_{47}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{72}$, $A_{51}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{66}$, $A_{52}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{52}$, $A_{54}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{59}$, $A_{60}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{73}$, $A_{67}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{75}$, $A_{86}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{97}$, $A_{87}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{89}$

A_6	$(3;2,6)$	$\underset{\mathrm{der}}{\sim}$	A_{51}	$(17)(264)(35)$	$\Rightarrow$	$A_7 \underset{\mathrm{der}}{\sim} A_{66}$
$A_6(*)$	$(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{56}	(46)	$\Rightarrow$	$A_7 \underset{\mathrm{der}}{\sim} A_{44}$
A_6	$(3;2,6),(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{21}	$(34)(56)$	$\Rightarrow$	$A_7 \underset{\mathrm{der}}{\sim} A_{39}$
A_8	$(4;3,6)$	$\underset{\mathrm{der}}{\sim}$	A_{47}	$(17)(2354)$	$\Rightarrow$	$A_8 \underset{\mathrm{der}}{\sim} A_{72}$
A_8	$(2;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{66}	$(16)(2435)$	$\Rightarrow$	$A_8 \underset{\mathrm{der}}{\sim} A_{51}$
$A_8(*)$	$(3;2,7)$	$\underset{\mathrm{der}}{\sim}$	A_{75}	$(167)(24)(35)$	$\Rightarrow$	$A_8 \underset{\mathrm{der}}{\sim} A_{67}$
$A_8(*)$	$(3;2,7),(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{28}	(162437)	$\Rightarrow$	$A_8 \underset{\mathrm{der}}{\sim} A_{26}$
A_{17}	$(3;2,6)$	$\underset{\mathrm{der}}{\sim}$	A_{86}	(3456)	$\Rightarrow$	$A_{36} \underset{\mathrm{der}}{\sim} A_{97}$
A_{17}	$(4;3)$	$\underset{\mathrm{der}}{\sim}$	A_{52}	$(47)(56)$	$\Rightarrow$	$A_{36} \underset{\mathrm{der}}{\sim} A_{52}$
A_{19}	$(3;2,6)$	$\underset{\mathrm{der}}{\sim}$	A_{87}	(345)	$\Rightarrow$	$A_{23} \underset{\mathrm{der}}{\sim} A_{89}$
A_{19}	$(3;2,6),(5;4,7)$	$\underset{\mathrm{der}}{\sim}$	A_{27}	$(34)(56)$	$\Rightarrow$	$A_{23} \underset{\mathrm{der}}{\sim} A_{29}$
A_{23}	$(6;5)$	$\underset{\mathrm{der}}{\sim}$	A_{44}	(467)	$\Rightarrow$	$A_{19} \underset{\mathrm{der}}{\sim} A_{56}$
A_{23}	$(6;5),(7;5)$	$\underset{\mathrm{der}}{\sim}$	A_{17}	(576)	$\Rightarrow$	$A_{19} \underset{\mathrm{der}}{\sim} A_{36}$
A_{26}	$(5;4),(7;4)$	$\underset{\mathrm{der}}{\sim}$	A_{37}	(1653247)	$\Rightarrow$	$A_{28} \underset{\mathrm{der}}{\sim} A_{37}$
A_{39}	$(2;1,4),(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{60}	(152436)	$\Rightarrow$	$A_{21} \underset{\mathrm{der}}{\sim} A_{73}$
A_{54}	$(2;1,4)$	$\underset{\mathrm{der}}{\sim}$	A_{73}	(176425)	$\Rightarrow$	$A_{59} \underset{\mathrm{der}}{\sim} A_{60}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.4 Polynomial $4(x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x - 1)$

$A_{14}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{31}$, $A_{15}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{22}$, $A_{46}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{57}$

A_{15}	$(6;5,7)$	$\underset{\mathrm{der}}{\sim}$	A_{22}	$(1735)(246)$		
A_{31}	$(5;2,7)$	$\underset{\mathrm{der}}{\sim}$	A_{15}	(56)	$\Rightarrow$	$A_{14} \underset{\mathrm{der}}{\sim} A_{22}$
$A_{46}(*)$	$(2;1,4,5)$	$\underset{\mathrm{der}}{\sim}$	A_{31}	(134)	$\Rightarrow$	$A_{57} \underset{\mathrm{der}}{\sim} A_{14}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.5 Polynomial $4(x^7 + x^6 - x^5 - x^4 + x^3 + x^2 - x - 1)$

$A_{45}^{\mathrm{op}} = A_{50}$

A_{50}	$(3;2,5,7)$	$\underset{\mathrm{der}}{\sim}$	A_{45}	(3476)

B.6 Polynomial $4(x^7 + x^5 - x^4 + x^3 - x^2 - 1)$

$A_9^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_9$, $A_{10}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{10}$, $A_{30}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{43}$, $A_{33}^{\mathrm{op}} = A_{34}$, $A_{40}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{40}$, $A_{48}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{80}$, $A_{58}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{76}$, $A_{61}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{69}$, $A_{63}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{64}$, $A_{68}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{68}$, $A_{70}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{78}$, $A_{77}^{\mathrm{op}} = A_{77}$, $A_{85}^{\mathrm{op}} = A_{99}$, $A_{88}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{91}$, $A_{92}^{\mathrm{op}} = A_{100}$, $A_{101}^{\mathrm{op}} = A_{103}$, $A_{109}^{\mathrm{op}} = A_{109}$, $A_{110}^{\mathrm{op}} = A_{111}$

A_9	$(3;2,7)$	$\underset{\mathrm{der}}{\sim}$	A_{69}	$(34)(576)$	$\Rightarrow$	$A_9 \underset{\mathrm{der}}{\sim} A_{61}$
A_{10}	$(2;1,6)$	$\underset{\mathrm{der}}{\sim}$	A_{78}	(1724)	$\Rightarrow$	$A_{10} \underset{\mathrm{der}}{\sim} A_{70}$
A_{10}	$(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{63}	(456)	$\Rightarrow$	$A_{10} \underset{\mathrm{der}}{\sim} A_{64}$
A_{10}	$(2;1,6),(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{111}	$(17)(2536)$	$\Rightarrow$	$A_{10} \underset{\mathrm{der}}{\sim} A_{110}$
$A_{30}(*)$	$(2;1,3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{48}	(13)	$\Rightarrow$	$A_{43} \underset{\mathrm{der}}{\sim} A_{80}$
A_{30}	$(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{58}	$(34)(567)$	$\Rightarrow$	$A_{43} \underset{\mathrm{der}}{\sim} A_{76}$
$A_{33}(*)$	$(2;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{103}	$(1724)(56)$	$\Rightarrow$	$A_{34} \underset{\mathrm{der}}{\sim} A_{101}$
A_{33}	$(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{88}	$(45)(67)$	$\Rightarrow$	$A_{34} \underset{\mathrm{der}}{\sim} A_{91}$
$A_{33}(*)$	$(2;1,5),(4;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{100}	$(164)(2573)$	$\Rightarrow$	$A_{34} \underset{\mathrm{der}}{\sim} A_{92}$
A_{33}	$(6;5)$	$\underset{\mathrm{der}}{\sim}$	A_{78}	$(35)(46)$	$\Rightarrow$	$A_{34} \underset{\mathrm{der}}{\sim} A_{70}$
A_{33}	$(4;3,7),(6;5)$	$\underset{\mathrm{der}}{\sim}$	A_{61}	$(45)(67)$	$\Rightarrow$	$A_{34} \underset{\mathrm{der}}{\sim} A_{69}$
A_{34}	$(3;2,7)$	$\underset{\mathrm{der}}{\sim}$	A_{99}	(475)	$\Rightarrow$	$A_{33} \underset{\mathrm{der}}{\sim} A_{85}$
A_{34}	$(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{48}	(3567)	$\Rightarrow$	$A_{33} \underset{\mathrm{der}}{\sim} A_{80}$
$A_{40}(*)$	$(2;1,6)$	$\underset{\mathrm{der}}{\sim}$	A_{92}	(1724)	$\Rightarrow$	$A_{40} \underset{\mathrm{der}}{\sim} A_{100}$
A_{68}	$(6;2)$	$\underset{\mathrm{der}}{\sim}$	A_{30}	(176543)	$\Rightarrow$	$A_{68} \underset{\mathrm{der}}{\sim} A_{43}$
A_{77}	$(2;1,3)$	$\underset{\mathrm{der}}{\sim}$	A_{110}	(1743526)	$\Rightarrow$	$A_{77} \underset{\mathrm{der}}{\sim} A_{111}$
A_{109}	$(3;2,7)$	$\underset{\mathrm{der}}{\sim}$	A_{63}	$(17456)(23)$	$\Rightarrow$	$A_{109} \underset{\mathrm{der}}{\sim} A_{64}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.7 Polynomial $4(x^7 + x^5 - 2x^4 + 2x^3 - x^2 - 1)$

$A_{38}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{41}$, $A_{71}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{71}$, $A_{95}^{\mathrm{op}} = A_{98}$

A_{38}	$(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{71}	(57)	$\Rightarrow$	$A_{41} \underset{\mathrm{der}}{\sim} A_{71}$
$A_{41}(*)$	$(2;1,6)$	$\underset{\mathrm{der}}{\sim}$	A_{95}	(15724)	$\Rightarrow$	$A_{38} \underset{\mathrm{der}}{\sim} A_{98}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.8 Polynomial $5(x^7 + x^5 - x^4 + x^3 - x^2 - 1)$

$A_{11}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{11}$, $A_{42}^{\mathrm{op}} = A_{42}$, $A_{65}^{\mathrm{op}} = A_{83}$, $A_{79}^{\mathrm{op}} = A_{82}$, $A_{81}^{\mathrm{op}} = A_{81}$, $A_{90}^{\mathrm{op}} = A_{105}$, $A_{94}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{94}$, $A_{102}^{\mathrm{op}} = A_{106}$, $A_{107}^{\mathrm{op}} = A_{108}$, $A_{112}^{\mathrm{op}} = A_{112}$

$A_{94}(*)$	$(2;1,4,7)$	$\underset{\mathrm{der}}{\sim}$	A_{106}	$(175)(23)$	$\Rightarrow$	$A_{94} \underset{\mathrm{der}}{\sim} A_{102}$
A_{94}	$(3;2),(5;4),(7;6)$	$\underset{\mathrm{der}}{\sim}$	A_{11}	$(37)(46)$		
A_{106}	$(1;3)$	$\underset{\mathrm{der}}{\sim}$	A_{81}	$(12)(4567)$	$\Rightarrow$	$A_{102} \underset{\mathrm{der}}{\sim} A_{81}$
A_{106}	$(4;3)$	$\underset{\mathrm{der}}{\sim}$	A_{112}	(1574)	$\Rightarrow$	$A_{102} \underset{\mathrm{der}}{\sim} A_{112}$
A_{108}	$(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{105}	(37654)	$\Rightarrow$	$A_{107} \underset{\mathrm{der}}{\sim} A_{90}$
A_{108}	$(1;2),(5;4)$	$\underset{\mathrm{der}}{\sim}$	A_{82}	$(1732)(46)$	$\Rightarrow$	$A_{107} \underset{\mathrm{der}}{\sim} A_{79}$
A_{108}	$(3;2),(5;4),(7;1,6)$	$\underset{\mathrm{der}}{\sim}$	A_{11}	$(27)(36)(45)$	$\Rightarrow$	$A_{107} \underset{\mathrm{der}}{\sim} A_{11}$
A_{83}	$(7;3)$	$\underset{\mathrm{der}}{\sim}$	A_{42}	$(16)(27)$	$\Rightarrow$	$A_{65} \underset{\mathrm{der}}{\sim} A_{42}$
A_{83}	$(5;4),(7;3)$	$\underset{\mathrm{der}}{\sim}$	A_{79}	(16275)	$\Rightarrow$	$A_{65} \underset{\mathrm{der}}{\sim} A_{82}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

B.9 Polynomial $6(x^7 + x^6 - x^4 + x^3 - x - 1)$

$A_{24}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{32}$, $A_{49}^{\mathrm{op}} = A_{74}$, $A_{55}^{\mathrm{op}} = A_{62}$, $A_{84}^{\mathrm{op}} = A_{96}$, $A_{93}^{\mathrm{op}} = A_{93}$

$A_{32}(*)$	$(2;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{93}	$(135)(67)$	$\Rightarrow$	$A_{24} \underset{\mathrm{der}}{\sim} A_{93}$
$A_{24}(*)$	$(2;1,5),(4;3)$	$\underset{\mathrm{der}}{\sim}$	A_{55}	$(1726)(35)$	$\Rightarrow$	$A_{32} \underset{\mathrm{der}}{\sim} A_{62}$
$A_{24}(*)$	$(2;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{96}	$(1726)(345)$	$\Rightarrow$	$A_{32} \underset{\mathrm{der}}{\sim} A_{84}$

A_{49}	$(2;1,5),(4;3)$	$\underset{\text{der}}{\sim}$	A_{24}	(35)	$\Rightarrow$	$A_{74} \underset{\text{der}}{\sim} A_{32}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

C Cluster-tilted algebras of type E_8

$x^8 - x^7 + x^5 - x^4 + x^3 - x + 1$	
algebra KQ/I	quiver Q
A_1	$(1,2),(2,3),(4,3),(5,4),(6,5),(7,6),(8,3)$

$2(x^8 - x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 + 1)$	
algebra KQ/I	quiver Q
A_2	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,5),(8,5)$
A_{19}	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,4),(6,7),(6,8),(7,5)$
A_{28}	$(1,2),(2,3),(3,4),(4,5),(5,6),(5,8),(6,4),(6,7),(7,5)$

$2(x^8 - x^6 + x^5 + x^3 - x^2 + 1)$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_3	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(6,4),(7,4),(8,7)$	A_4	$(1,2),(2,3),(4,3),(5,3),$ $(5,6),(6,7),(7,5),(8,7)$
A_5	$(1,2),(2,3),(4,3),(5,3),$ $(6,5),(6,7),(7,8),(8,6)$	A_6	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(5,8),(6,3),(7,6)$
A_7	$(1,2),(2,3),(4,3),(5,3),$ $(5,6),(6,7),(6,8),(7,5)$	A_{10}	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(5,7),(6,3),(7,8)$
A_{23}	$(1,2),(2,3),(4,3),(4,5),$ $(4,7),(5,6),(6,4),(7,6),(8,7)$	A_{31}	$(1,2),(3,2),(3,4),(3,6),$ $(4,5),(5,3),(5,8),(6,5),(7,4)$
A_{35}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,4),(7,4),(7,8)$	A_{46}	$(2,1),(3,2),(3,4),(4,5),$ $(4,6),(4,8),(5,3),(6,3),(7,5)$

$2(x^8 - 2x^6 + 4x^5 - 4x^4 + 4x^3 - 2x^2 + 1)$	
algebra KQ/I	quiver Q
A_{25}	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,4),(6,7),(7,5),(8,7)$

$3(x^8 + x^4 + 1)$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_8	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,4),(7,8)$	A_9	$(1,2),(2,3),(4,3),(4,5),$ $(5,6),(6,7),(7,4),(8,5)$
A_{12}	$(1,2),(3,2),(3,4),(4,5),$ $(5,6),(5,8),(6,3),(7,4)$	A_{14}	$(2,1),(3,2),(3,4),(4,5),$ $(5,6),(5,8),(6,3),(7,6)$
A_{17}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(7,3),(8,4)$	A_{26}	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(5,6),(6,7),(6,8),(7,4)$
A_{30}	$(1,2),(2,3),(4,3),(4,5),$ $(5,6),(6,7),(7,4),(7,8),(8,6)$	A_{33}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{34}	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(5,6),(6,7),(7,4),(8,7)$	A_{43}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,6),(6,4),(7,8),(8,6)$
A_{44}	$(1,2),(2,3),(3,4),(4,5),$ $(4,8),(5,3),(5,6),(6,7),(7,4)$	A_{47}	$(1,2),(3,2),(3,4),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5)$
A_{53}	$(1,2),(2,3),(3,4),(3,7),$ $(4,5),(5,6),(6,3),(7,6),(8,5)$	A_{60}	$(1,2),(2,3),(4,3),(4,5),$ $(5,6),(5,8),(6,7),(7,4),(8,4)$
A_{61}	$(2,1),(3,2),(3,4),(4,5),$ $(4,6),(5,3),(6,7),(6,8),(7,3)$	A_{66}	$(1,2),(2,3),(3,4),(4,5),$ $(5,3),(5,6),(5,8),(6,7),(7,4)$
A_{67}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,3),(7,4),(8,5)$	A_{76}	$(2,1),(2,3),(3,4),(3,6),$ $(4,5),(5,2),(6,2),(7,3),(8,7)$
A_{80}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,6),(6,3),(7,3),(8,4)$	A_{84}	$(1,2),(2,3),(3,4),(3,8),$ $(4,2),(4,5),(5,6),(6,3),(7,4)$
A_{92}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,6),(6,4),(6,7),(7,5),(8,6)$	A_{94}	$(1,2),(2,3),(3,4),(4,5),(5,3),$ $(5,6),(6,4),(6,7),(7,5),(8,6)$
A_{100}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,3),(5,6),(6,4),(6,7),(8,6)$	A_{102}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,3),(5,6),(6,4),(6,8),(8,5)$
A_{109}	$(1,2),(2,3),(4,3),(4,5),(5,6),$ $(5,7),(6,4),(7,4),(7,8),(8,5)$	A_{110}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(8,5)$
A_{111}	$(1,2),(2,3),(2,5),(3,4),(4,2),$ $(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{123}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,6),(6,4),(7,6),(7,8),(8,4)$
A_{131}	$(1,2),(2,3),(3,4),(3,5),(5,2),$ $(5,6),(6,3),(6,7),(7,5),(7,8)$	A_{132}	$(1,2),(2,3),(3,4),(3,5),(5,2),$ $(5,6),(6,7),(7,3),(7,8),(8,6)$
A_{144}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(4,8),(6,3),(6,7),(7,4),(8,7)$	A_{148}	$(2,1),(3,2),(3,4),(4,5),(4,7),$ $(5,3),(5,6),(6,4),(7,6),(8,5)$
A_{149}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,3),(6,7),(7,5),(8,4)$	A_{154}	$(1,2),(3,2),(3,4),(4,5),(4,6),$ $(4,8),(5,3),(6,3),(6,7),(7,4)$
A_{163}	$(1,2),(2,3),(3,4),(3,8),(4,5),$	A_{169}	$(1,2),(2,3),(3,4),(3,7),(4,5),$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
	$(5,3),(5,6),(5,7),(7,8),(8,5)$		$(4,6),(5,3),(6,3),(7,6),(8,4)$
A_{171}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5)$	A_{173}	$(1,2),(2,3),(3,4),(3,5),(3,7),$ $(5,2),(5,6),(6,3),(7,6),(8,7)$
A_{187}	$(1,2),(3,2),(3,4),(4,5),(5,3),$ $(5,6),(5,7),(7,4),(7,8),(8,5)$	A_{196}	$(2,1),(2,3),(3,4),(3,6),(3,7),$ $(4,2),(4,5),(5,3),(6,2),(7,8)$
A_{206}	$(2,1),(2,3),(2,5),(3,4),(4,2),$ $(5,4),(5,6),(6,2),(7,5),(8,6)$	A_{218}	$(2,1),(2,3),(3,4),(3,8),(4,2),$ $(4,5),(4,6),(6,3),(6,7),(7,4)$
A_{221}	$(1,2),(2,3),(2,4),(4,1),(4,5),$ $(5,6),(6,2),(6,7),(7,5),(8,5)$	A_{222}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,2),(5,6),(5,7),(7,4),(8,3)$
A_{232}	$(1,2),(2,3),(3,4),(4,5),(5,3),$ $(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$	A_{242}	$(1,2),(3,2),(3,4),(4,5),(4,8),$ $(5,3),(5,6),(6,4),(6,7),(7,8),(8,6)$
A_{247}	$(1,2),(2,3),(3,4),(3,7),(4,5),$ $(5,3),(5,6),(6,7),(7,5),(7,8),(8,6)$	A_{272}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,5),(8,6)$
A_{275}	$(2,1),(2,3),(3,4),(3,7),(4,2),$ $(4,5),(5,3),(5,6),(6,7),(7,5),(7,8)$	A_{277}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,3),(6,3),(6,7),(7,4),(7,8),(8,6)$
A_{305}	$(2,1),(2,3),(2,6),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,5),(8,5)$		

$\mathbf{4(x^8+x^7-x^6+x^5+x^3-x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{20}	$(1,2),(2,3),(3,4),(4,5),$ $(4,7),(5,3),(6,4),(7,8),(8,4)$	A_{21}	$(1,2),(2,3),(3,1),(4,2),$ $(5,2),(5,6),(6,7),(7,5),(8,7)$
A_{22}	$(1,2),(2,3),(3,1),(4,2),$ $(5,2),(6,5),(6,7),(7,8),(8,6)$	A_{27}	$(2,1),(2,3),(3,4),(4,2),$ $(4,6),(5,4),(6,7),(7,4),(8,3)$
A_{29}	$(2,1),(2,3),(3,4),(4,2),$ $(4,5),(5,7),(6,5),(7,8),(8,5)$	A_{36}	$(1,2),(2,3),(3,4),(4,2),$ $(5,3),(5,6),(5,7),(7,8),(8,5)$
A_{37}	$(1,2),(2,3),(2,4),(2,5),$ $(3,1),(6,5),(6,7),(7,8),(8,6)$	A_{41}	$(2,1),(3,2),(3,4),(4,5),$ $(5,3),(5,7),(6,5),(7,8),(8,5)$
A_{49}	$(1,2),(2,3),(2,4),(2,5),$ $(3,1),(5,6),(6,7),(6,8),(7,5)$	A_{52}	$(1,2),(2,3),(3,1),(4,2),$ $(5,4),(5,7),(6,5),(7,8),(8,5)$
A_{89}	$(2,1),(2,3),(2,5),(3,4),(4,2),$ $(5,4),(6,5),(6,7),(7,8),(8,6)$	A_{90}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(6,3),(6,7),(6,8),(7,5)$
A_{98}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,4),(6,7),(6,8),(7,5)$	A_{105}	$(1,2),(2,3),(2,4),(3,1),(4,5),$ $(5,6),(5,8),(6,4),(6,7),(7,5)$
A_{106}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,5),(8,7)$	A_{122}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,8),(8,6)$
A_{124}	$(2,1),(2,3),(2,5),(3,4),(4,2),$ $(5,4),(5,6),(6,7),(7,8),(8,6)$	A_{142}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(7,4),(7,8),(8,5)$

$\mathbf{4(x^8+x^7-x^6+2x^4-x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{24}	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(6,3),(6,7),(7,8),(8,6)$	A_{32}	$(1,2),(2,3),(3,5),(4,3),$ $(5,6),(5,7),(6,3),(7,8),(8,5)$
A_{93}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(5,3),(6,7),(7,8),(8,6)$	A_{107}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(5,3),(6,7),(7,4),(7,8)$
A_{113}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(4,6),(4,7),(5,3),(6,3),(8,6)$	A_{120}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,7),(6,3),(7,3),(7,8)$
A_{121}	$(1,2),(2,3),(2,4),(3,1),(4,5),$ $(5,6),(5,7),(6,4),(7,4),(7,8)$	A_{137}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(6,4),(6,7),(7,8),(8,6)$
A_{146}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,6),(5,3),(6,7),(6,8),(7,4)$	A_{152}	$(1,2),(2,3),(3,1),(3,4),(3,7),$ $(4,5),(5,3),(5,6),(7,5),(8,4)$
A_{153}	$(1,2),(2,3),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(6,8),(8,3)$	A_{155}	$(1,2),(2,3),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(7,8),(8,6)$

$\mathbf{4(x^8+x^7-2x^6+2x^5+2x^3-2x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{95}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{96}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(6,3),(6,7),(7,5),(8,7)$
A_{116}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{119}	$(2,1),(2,3),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,7),(7,8),(8,6)$

$\mathbf{4(x^8+x^6-x^5+2x^4-x^3+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{11}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,8),(8,4)$	A_{13}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(7,3),(8,5)$
A_{16}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,7),(6,8),(7,3)$	A_{40}	$(2,1),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,4),(8,7)$
A_{42}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(6,3),(6,7),(7,8),(8,5)$	A_{54}	$(2,1),(2,3),(3,4),(4,5),$ $(4,6),(5,2),(6,7),(7,3),(8,6)$
A_{55}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,3),(7,8),(8,4)$	A_{58}	$(1,2),(2,3),(3,4),(3,8),$ $(4,2),(4,5),(5,6),(6,7),(7,3)$
A_{72}	$(1,2),(2,3),(3,4),(4,2),$ $(4,5),(4,6),(6,7),(7,8),(8,3)$	A_{85}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,8),(6,2),(6,7),(7,5)$
A_{87}	$(1,2),(2,3),(2,7),(3,4),$ $(4,5),(5,6),(5,8),(6,2),(7,6)$	A_{99}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(5,8),(6,3),(7,4)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{103}	$(1,2),(2,3),(3,4),(4,5),(5,3),(5,6),(6,4),(6,7),(7,8),(8,5)$	A_{104}	$(1,2),(2,3),(3,4),(3,8),(4,5),(5,3),(5,6),(6,7),(7,4),(8,5)$
A_{112}	$(1,2),(2,3),(3,4),(4,5),(5,6),(5,7),(6,3),(7,4),(7,8),(8,5)$	A_{126}	$(1,2),(2,3),(3,4),(3,8),(4,2),(4,5),(4,6),(6,7),(7,3),(8,7)$
A_{127}	$(2,1),(2,3),(2,8),(3,4),(4,5),(5,6),(5,7),(6,2),(7,4),(8,6)$	A_{128}	$(1,2),(2,3),(3,4),(3,6),(4,5),(5,3),(5,8),(6,2),(6,7),(7,5)$
A_{129}	$(1,2),(2,3),(3,4),(4,5),(5,6),(5,8),(6,3),(6,7),(7,5),(8,7)$	A_{133}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),(5,6),(6,7),(6,8),(7,4)$
A_{134}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,6),(6,3),(6,7),(7,8),(8,5)$	A_{135}	$(1,2),(2,3),(3,4),(4,5),(5,6),(6,3),(6,7),(7,5),(7,8),(8,6)$
A_{136}	$(1,2),(2,3),(3,4),(4,5),(4,6),(5,3),(6,3),(6,7),(7,8),(8,4)$	A_{143}	$(1,2),(2,3),(3,4),(3,5),(5,2),(5,6),(6,7),(6,8),(7,3),(8,5)$
A_{150}	$(2,1),(2,3),(3,4),(3,6),(4,2),(4,5),(5,3),(6,7),(7,5),(8,7)$	A_{151}	$(1,2),(2,3),(2,5),(3,4),(4,2),(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{162}	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,3),(5,6),(6,4),(7,8),(8,6)$	A_{170}	$(1,2),(2,3),(3,4),(4,5),(4,6),(5,2),(6,7),(7,3),(7,8),(8,6)$
A_{172}	$(2,1),(2,3),(3,4),(3,5),(4,2),(5,6),(6,7),(7,2),(7,8),(8,6)$	A_{175}	$(2,1),(2,3),(3,4),(4,5),(5,2),(5,6),(6,7),(7,4),(7,8),(8,6)$
A_{176}	$(1,2),(2,3),(3,4),(4,5),(5,2),(5,6),(6,4),(6,7),(6,8),(7,5)$	A_{177}	$(1,2),(2,3),(3,4),(3,5),(4,2),(5,2),(5,6),(6,7),(7,3),(7,8)$
A_{182}	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,6),(6,4),(7,6),(7,8),(8,3)$	A_{185}	$(2,1),(2,3),(3,4),(3,5),(4,2),(5,6),(6,7),(6,8),(7,2),(8,5)$
A_{186}	$(1,2),(2,3),(3,4),(3,7),(3,8),(4,2),(4,5),(5,6),(6,3),(7,6)$	A_{192}	$(1,2),(2,3),(3,4),(3,6),(4,2),(4,5),(5,3),(6,7),(7,5),(8,6)$
A_{193}	$(1,2),(2,3),(3,4),(4,2),(4,5),(4,6),(6,7),(6,8),(7,4),(8,3)$	A_{207}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),(5,6),(6,7),(7,4),(8,7)$
A_{208}	$(1,2),(2,3),(3,4),(4,5),(4,6),(5,3),(6,2),(6,7),(7,4),(8,4)$	A_{224}	$(1,2),(2,3),(3,4),(4,5),(5,3),(5,6),(5,8),(6,4),(6,7),(7,5),(8,7)$
A_{225}	$(1,2),(2,3),(3,4),(3,8),(4,5),(5,3),(5,6),(6,4),(6,7),(7,5),(8,5)$	A_{226}	$(2,1),(2,3),(3,4),(3,5),(3,7),(4,2),(5,2),(5,6),(6,3),(7,6),(8,7)$
A_{227}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,6),(5,8),(6,3),(6,7),(7,5),(8,7)$	A_{231}	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,3),(5,6),(6,4),(7,6),(7,8),(8,4)$
A_{237}	$(1,2),(2,3),(2,8),(3,4),(4,2),(4,5),(5,3),(5,6),(6,4),(6,7),(8,4)$	A_{238}	$(1,2),(2,3),(2,8),(3,4),(4,2),(4,5),(5,6),(6,3),(6,7),(7,5),(8,4)$
A_{239}	$(2,1),(3,2),(3,4),(4,5),(4,7),(4,8),(5,3),(5,6),(6,4),(7,6),(8,3)$	A_{240}	$(2,1),(2,3),(3,4),(3,5),(4,2),(5,2),(5,6),(6,7),(7,3),(7,8),(8,6)$
A_{243}	$(1,2),(2,3),(3,4),(4,1),(4,5),(5,3),(5,6),(6,4),(6,7),(7,5),(8,7)$	A_{258}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,6),(6,3),(6,7),(7,5),(7,8),(8,6)$
A_{273}	$(1,2),(2,3),(3,4),(4,5),(4,8),(5,2),(5,6),(6,4),(6,7),(7,8),(8,6)$	A_{276}	$(2,1),(2,3),(3,4),(4,5),(4,8),(5,3),(5,6),(6,4),(6,7),(7,5),(8,2)$
A_{282}	$(1,2),(2,3),(2,7),(3,4),(4,2),(4,5),(5,6),(6,7),(7,4),(7,8),(8,6)$	A_{286}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),(5,6),(6,4),(6,7),(7,8),(8,5)$
A_{304}	$(2,1),(2,3),(3,4),(3,5),(4,2),(5,2),(5,6),(6,7),(6,8),(7,5),(8,3)$	A_{311}	$(1,2),(2,3),(2,8),(3,4),(4,2),(4,5),(5,6),(6,4),(6,7),(7,5),(8,6)$
A_{324}	$(2,1),(2,3),(3,4),(3,5),(4,2),(5,6),(5,7),(6,2),(7,3),(7,8),(8,5)$	A_{333}	$(1,2),(2,3),(3,4),(3,6),(4,2),(4,5),(5,3),(6,5),(6,7),(7,3),(7,8),(8,6)$
A_{337}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$	A_{338}	$(1,2),(2,3),(3,4),(3,7),(4,2),(4,5),(5,3),(5,6),(6,7),(7,5),(7,8),(8,6)$
A_{342}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,3),(5,6),(5,8),(6,7),(7,5),(8,4),(8,7)$	A_{343}	$(1,2),(2,3),(2,5),(3,1),(3,4),(4,2),(5,1),(5,6),(6,2),(6,7),(7,5),(8,7)$
A_{352}	$(1,2),(2,3),(3,1),(3,4),(4,5),(5,2),(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$	A_{361}	$(1,2),(2,3),(3,4),(3,7),(3,8),(4,2),(4,5),(5,3),(5,6),(6,7),(7,5),(8,2)$
A_{363}	$(1,2),(2,3),(3,1),(3,4),(4,2),(4,5),(4,8),(5,6),(5,7),(7,4),(8,3),(8,7)$	A_{366}	$(2,1),(2,3),(2,8),(3,4),(4,2),(4,5),(5,6),(5,8),(6,4),(6,7),(7,5),(8,4)$
A_{370}	$(1,2),(2,3),(3,1),(3,4),(4,5),(4,8),(5,2),(5,6),(6,4),(6,7),(7,8),(8,6)$	A_{388}	$(1,2),(2,3),(2,4),(3,1),(4,1),(4,5),(5,2),(5,6),(6,4),(6,7),(7,5),(7,8),(8,6)$

$\mathbf{4(x^8+x^6-2x^5+4x^4-2x^3+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{48}	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,6),(6,2),(7,3),(8,7)$	A_{70}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,6),(5,8),(6,7),(7,3)$
A_{118}	$(1,2),(2,3),(3,4),(4,5),(4,7),(5,2),(5,6),(6,4),(7,3),(8,7)$	A_{160}	$(1,2),(2,3),(3,4),(3,7),(4,2),(4,5),(5,6),(6,3),(7,6),(8,5)$
A_{161}	$(1,2),(2,3),(3,1),(3,4),(4,5),(5,6),(6,2),(6,7),(7,5),(8,4)$	A_{278}	$(1,2),(2,3),(2,8),(3,4),(3,7),(4,1),(4,5),(5,3),(5,6),(7,2),(8,7)$
A_{302}	$(1,2),(2,3),(3,4),(4,2),(4,5),(5,6),(5,7),(6,3),(7,4),(7,8),(8,5)$		

$\mathbf{5(x^8+x^6+x^4+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{18}	$(1,2),(2,3),(3,4),(4,5),(5,6),(5,8),(6,7),(7,2)$	A_{51}	$(2,1),(2,3),(3,4),(4,5),(5,6),(5,8),(6,7),(7,2),(8,4)$
A_{65}	$(1,2),(2,3),(3,4),(4,5),(4,8),(5,6),(6,7),(7,2),(8,3)$	A_{69}	$(2,1),(2,3),(3,4),(4,5),(5,6),(6,7),(6,8),(7,2),(8,5)$
A_{71}	$(2,1),(2,3),(3,4),(4,5),(5,6),(6,7),(7,2),(7,8),(8,6)$	A_{74}	$(1,2),(2,3),(3,4),(3,7),(3,8),(4,5),(5,6),(6,1),(7,2)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{77}	$(1,2),(2,3),(3,4),(4,5),$ $(4,6),(5,2),(6,7),(7,8),(8,3)$	A_{78}	$(1,2),(2,3),(2,8),(3,4),$ $(3,7),(4,5),(5,6),(6,1),(7,2)$
A_{83}	$(1,2),(2,3),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,8),(8,4)$	A_{86}	$(2,1),(2,3),(3,4),(4,5),$ $(5,2),(5,6),(6,7),(7,8),(8,4)$
A_{125}	$(2,1),(2,3),(3,4),(4,5),(5,6),$ $(5,7),(6,2),(6,8),(7,4),(8,5)$	A_{140}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(6,7),(7,8),(8,4)$
A_{159}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,6),(5,8),(6,2),(7,3),(8,4)$	A_{165}	$(2,1),(2,3),(2,7),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(7,8),(8,6)$
A_{166}	$(2,1),(2,3),(3,4),(3,7),(4,5),$ $(4,8),(5,6),(6,2),(7,2),(8,3)$	A_{174}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,2),(5,6),(6,7),(7,8),(8,3)$
A_{178}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(6,2),(6,7),(7,8),(8,5)$	A_{181}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,2),(6,7),(6,8),(7,4),(8,3)$
A_{183}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,8),(8,6)$	A_{199}	$(1,2),(2,3),(3,4),(3,7),(4,5),$ $(5,6),(6,3),(7,2),(7,8),(8,6)$
A_{200}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,2),(6,3),(6,7),(7,8),(8,4)$	A_{201}	$(1,2),(2,3),(2,4),(2,8),(4,1),$ $(4,5),(5,6),(6,7),(7,2),(8,7)$
A_{202}	$(2,1),(2,3),(3,4),(4,5),(4,8),$ $(5,2),(5,6),(6,7),(7,4),(8,7)$	A_{203}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(5,6),(5,7),(6,3),(7,8),(8,4)$
A_{204}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,8),(8,4)$	A_{212}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(6,7),(7,4),(8,7)$
A_{213}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(4,6),(5,2),(6,7),(7,3),(8,7)$	A_{214}	$(1,2),(2,3),(3,4),(4,5),(4,8),$ $(5,6),(5,7),(6,2),(7,4),(8,7)$
A_{216}	$(1,2),(2,3),(3,4),(4,5),(5,6),$ $(5,7),(6,2),(7,4),(7,8),(8,5)$	A_{219}	$(1,2),(2,3),(3,4),(4,5),(5,2),$ $(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{220}	$(1,2),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,8),(6,7),(7,3),(8,2)$	A_{223}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,6),(6,3),(6,7),(7,8),(8,5)$
A_{234}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(5,3),(5,6),(5,7),(7,8),(8,2),(8,5)$	A_{241}	$(1,2),(1,4),(2,3),(3,1),(4,5),$ $(5,6),(5,8),(6,3),(6,7),(7,5),(8,7)$
A_{246}	$(1,2),(2,3),(2,8),(3,4),(4,2),$ $(4,5),(5,6),(5,7),(6,3),(7,4),(8,4)$	A_{249}	$(1,2),(2,3),(2,7),(3,1),(3,4),$ $(4,5),(5,2),(5,6),(6,4),(7,5),(8,3)$
A_{252}	$(1,2),(2,3),(3,4),(3,7),(4,2),$ $(4,5),(5,6),(6,3),(7,6),(7,8),(8,3)$	A_{260}	$(2,1),(2,3),(3,4),(3,5),(4,2),$ $(5,6),(5,7),(6,3),(7,2),(7,8),(8,5)$
A_{261}	$(1,2),(2,3),(3,4),(3,8),(4,2),$ $(4,5),(4,6),(6,3),(6,7),(7,4),(8,6)$	A_{262}	$(1,2),(2,3),(2,4),(2,5),(3,1),$ $(5,6),(5,7),(6,2),(7,1),(7,8),(8,5)$
A_{263}	$(1,2),(2,3),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,6),(7,8),(8,4)$	A_{265}	$(2,1),(2,3),(3,4),(3,5),(3,8),$ $(4,2),(5,2),(5,6),(6,7),(7,3),(8,7)$
A_{266}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,8),(5,6),(5,7),(6,3),(7,4),(8,7)$	A_{267}	$(1,2),(1,4),(2,3),(3,1),(4,5),$ $(4,8),(5,6),(5,7),(6,3),(7,4),(8,7)$
A_{274}	$(1,2),(2,3),(3,1),(3,4),(4,5),$ $(5,6),(5,7),(6,2),(7,4),(7,8),(8,5)$	A_{279}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,3),(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{281}	$(1,2),(1,8),(2,3),(3,1),(3,4),$ $(4,5),(5,6),(5,7),(6,4),(7,8),(8,3)$	A_{283}	$(1,2),(2,3),(3,4),(3,6),(4,1),$ $(4,5),(5,3),(6,5),(6,7),(7,3),(8,7)$
A_{285}	$(2,1),(2,3),(2,6),(3,4),(4,2),$ $(4,5),(5,6),(6,4),(6,7),(7,2),(8,7)$	A_{293}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(4,8),(5,2),(6,7),(7,4),(8,3),(8,7)$
A_{295}	$(1,2),(2,3),(2,7),(2,8),(3,1),$ $(3,4),(4,5),(5,2),(5,6),(6,4),(8,5)$	A_{296}	$(1,2),(2,3),(3,4),(4,2),(4,5),$ $(4,7),(5,3),(5,6),(6,4),(7,8),(8,6)$
A_{297}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,3),(5,6),(6,7),(7,4),(7,8),(8,6)$	A_{303}	$(1,2),(2,3),(3,4),(3,8),(4,1),$ $(4,5),(5,3),(5,6),(6,7),(7,5),(8,7)$
A_{306}	$(1,2),(2,3),(3,4),(3,8),(4,5),$ $(4,7),(5,1),(5,6),(6,4),(7,3),(8,7)$	A_{307}	$(1,2),(2,3),(2,4),(3,1),(4,5),$ $(4,7),(5,6),(6,1),(7,2),(7,8),(8,4)$
A_{310}	$(1,2),(1,5),(2,3),(3,4),(3,6),$ $(4,1),(5,4),(6,2),(6,7),(7,8),(8,3)$	A_{312}	$(1,2),(2,3),(3,4),(3,5),(3,7),$ $(4,2),(5,2),(5,6),(6,3),(7,8),(8,6)$
A_{314}	$(1,2),(2,3),(3,4),(3,6),(4,1),$ $(4,5),(5,3),(6,5),(6,7),(7,3),(8,6)$	A_{315}	$(1,2),(2,3),(3,1),(3,4),(4,2),$ $(4,5),(5,3),(5,6),(6,7),(7,8),(8,4)$
A_{318}	$(1,2),(2,3),(3,4),(4,5),(4,6),$ $(5,1),(6,3),(6,7),(7,4),(7,8),(8,6)$	A_{321}	$(2,1),(2,3),(2,7),(3,4),(4,2),$ $(4,5),(5,6),(5,8),(6,4),(7,6),(8,4)$
A_{322}	$(1,2),(2,3),(3,4),(4,1),(4,5),$ $(5,6),(6,3),(6,7),(7,5),(7,8),(8,6)$	A_{326}	$(1,2),(2,3),(3,4),(3,8),(4,1),$ $(4,5),(5,3),(5,6),(6,7),(7,8),(8,5)$
A_{327}	$(1,2),(2,3),(3,4),(3,5),(3,7),$ $(4,2),(5,6),(6,3),(7,6),(7,8),(8,2)$	A_{328}	$(1,2),(2,3),(3,1),(3,4),(4,2),$ $(4,5),(5,6),(6,3),(6,7),(7,8),(8,5)$
A_{335}	$(1,2),(1,4),(2,3),(3,1),(3,6),(4,3),$ $(4,5),(5,6),(6,4),(6,7),(7,3),(8,7)$	A_{340}	$(1,2),(2,3),(3,1),(3,4),(3,8),(4,2),$ $(4,5),(5,3),(5,6),(6,4),(6,7),(8,5)$
A_{345}	$(1,2),(2,3),(2,6),(3,4),(4,2),(4,5),$ $(5,6),(5,8),(6,4),(6,7),(7,5),(8,4)$	A_{346}	$(1,2),(2,3),(2,7),(3,1),(3,4),(4,2),$ $(4,5),(5,6),(5,8),(6,7),(7,4),(8,4)$
A_{348}	$(1,2),(2,3),(3,4),(3,8),(4,2),(4,5),$ $(5,3),(5,6),(6,4),(6,7),(7,5),(8,5)$	A_{349}	$(1,2),(2,3),(3,1),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,3),(7,8),(8,4)$
A_{350}	$(1,2),(2,3),(3,1),(3,4),(4,5),(4,8),$ $(5,2),(5,6),(6,4),(6,7),(7,5),(8,3)$	A_{351}	$(1,2),(2,3),(2,8),(3,4),(4,2),(4,5),$ $(4,7),(5,6),(6,4),(7,6),(7,8),(8,4)$
A_{353}	$(1,2),(2,3),(3,4),(3,5),(3,7),(4,2),$ $(5,2),(5,6),(6,3),(7,6),(7,8),(8,3)$	A_{354}	$(1,2),(2,3),(3,1),(3,4),(4,2),(4,5),$ $(5,3),(5,6),(6,7),(6,8),(7,4),(8,5)$
A_{355}	$(2,1),(2,3),(3,4),(3,5),(3,8),(4,2),$ $(5,2),(5,6),(6,3),(6,7),(7,5),(8,6)$	A_{356}	$(1,2),(2,3),(2,6),(3,1),(3,4),(4,2),$ $(4,5),(4,7),(5,6),(6,4),(7,3),(8,7)$
A_{357}	$(1,2),(2,3),(2,8),(3,1),(3,4),(4,5),$ $(4,7),(5,2),(5,6),(6,4),(7,6),(8,5)$	A_{360}	$(1,2),(2,3),(3,1),(3,4),(3,8),(4,5),$ $(5,3),(6,2),(6,7),(7,8),(8,5),(8,6)$
A_{362}	$(1,2),(2,3),(3,1),(3,4),(4,5),(4,7),$ $(5,2),(5,6),(6,4),(7,6),(7,8),(8,4)$	A_{364}	$(1,2),(2,3),(3,4),(3,7),(3,8),(4,1),$ $(4,5),(5,3),(5,6),(6,7),(7,5),(8,2)$
A_{365}	$(1,2),(2,3),(3,1),(3,4),(3,6),(4,2),$ $(4,5),(5,3),(6,7),(7,5),(7,8),(8,6)$	A_{367}	$(1,2),(2,3),(2,5),(3,1),(3,4),(4,2),$ $(5,4),(5,6),(6,7),(6,8),(7,2),(8,5)$

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{368}	(1,2), (2,3), (3,4), (3,6), (3,8), (4,2), (4,5), (5,3), (6,5), (6,7), (7,3), (8,7)	A_{369}	(1,2), (1,8), (2,3), (3,1), (3,4), (4,5), (4,8), (5,6), (6,7), (7,4), (8,3), (8,7)
A_{371}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,6), (5,2), (6,3), (6,7), (7,4), (7,8), (8,6)	A_{372}	(1,2), (2,3), (2,5), (2,7), (3,1), (3,4), (4,2), (5,1), (5,6), (6,2), (7,6), (7,8)
A_{373}	(1,2), (2,3), (2,4), (3,1), (4,1), (4,5), (5,6), (6,2), (6,7), (7,5), (7,8), (8,6)	A_{374}	(1,2), (2,3), (3,4), (3,7), (4,1), (4,5), (5,3), (5,6), (6,7), (6,8), (7,5), (8,5)
A_{375}	(1,2), (2,3), (3,1), (3,4), (3,8), (4,2), (4,5), (5,3), (5,6), (6,7), (7,5), (8,7)	A_{378}	(1,2), (2,3), (3,4), (3,7), (4,1), (4,5), (5,3), (5,6), (6,7), (7,5), (7,8), (8,6)
A_{379}	(2,1), (2,3), (2,6), (3,4), (4,2), (4,5), (5,6), (6,4), (6,7), (6,8), (7,5), (8,2)	A_{380}	(1,2), (2,3), (2,7), (3,4), (4,2), (4,5), (5,6), (5,7), (6,4), (7,4), (7,8), (8,2)
A_{381}	(1,2), (2,3), (3,1), (3,4), (3,8), (4,2), (4,5), (5,3), (5,6), (6,7), (7,8), (8,5)	A_{382}	(1,2), (2,3), (2,5), (3,1), (3,4), (4,2), (5,6), (6,4), (6,7), (7,5), (7,8), (8,6)
A_{383}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (5,6), (5,8), (6,3), (6,7), (7,5), (8,7)	A_{386}	(1,2), (2,3), (2,6), (3,1), (3,4), (4,2), (4,5), (5,6), (6,4), (6,7), (7,5), (7,8), (8,6)
A_{389}	(1,2), (2,3), (3,1), (3,4), (3,7), (4,2), (4,5), (5,3), (5,6), (6,7), (6,8), (7,5), (8,5)	A_{390}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (5,3), (5,6), (5,8), (6,4), (6,7), (7,5), (8,7)

$\boldsymbol{6(x^8+x^6+x^5+x^3+x^2+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{15}	(2,1), (2,3), (3,4), (4,5), (5,6), (6,7), (7,8), (8,2)	A_{88}	(1,2), (2,3), (3,4), (3,5), (4,1), (5,6), (6,7), (7,8), (8,2)
A_{179}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (5,6), (6,7), (7,8), (8,3)	A_{184}	(1,2), (2,3), (2,8), (3,4), (4,5), (5,6), (6,7), (7,2), (8,1), (8,7)
A_{205}	(1,2), (2,3), (3,4), (4,5), (4,8), (5,1), (5,6), (6,7), (7,4), (8,3)	A_{209}	(1,2), (2,3), (3,4), (3,5), (4,1), (5,2), (5,6), (6,7), (7,8), (8,3)
A_{211}	(1,2), (2,3), (3,4), (4,5), (4,7), (5,1), (5,6), (6,4), (7,8), (8,3)	A_{215}	(1,2), (2,3), (3,4), (3,6), (4,1), (4,5), (5,3), (6,7), (7,8), (8,5)
A_{268}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,6), (5,3), (6,2), (6,7), (7,8), (8,4)	A_{270}	(1,2), (2,3), (3,4), (4,5), (4,7), (4,8), (5,1), (5,6), (6,4), (7,6), (8,3)
A_{280}	(1,2), (1,7), (2,3), (3,1), (3,4), (4,5), (4,8), (5,6), (6,7), (7,3), (8,3)	A_{290}	(1,2), (2,3), (3,4), (3,6), (4,1), (4,5), (5,3), (5,7), (6,5), (7,4), (8,7)
A_{299}	(1,2), (2,3), (2,7), (3,1), (3,4), (4,5), (4,6), (5,2), (6,3), (7,8), (8,5)	A_{300}	(1,2), (2,3), (3,4), (3,6), (3,8), (4,1), (4,5), (5,3), (6,7), (7,5), (8,2)
A_{308}	(1,2), (2,3), (3,4), (3,6), (4,1), (4,5), (5,3), (6,7), (6,8), (7,5), (8,3)	A_{309}	(1,2), (2,3), (3,4), (3,8), (4,5), (4,6), (5,2), (6,3), (6,7), (7,4), (8,6)
A_{313}	(1,2), (2,3), (3,4), (4,5), (4,7), (5,1), (5,6), (6,4), (7,6), (7,8), (8,4)	A_{317}	(1,2), (2,3), (3,4), (4,5), (4,8), (5,2), (5,6), (6,4), (6,7), (7,5), (8,6)
A_{319}	(1,2), (2,3), (3,4), (3,8), (4,5), (5,3), (5,6), (6,7), (7,8), (8,2), (8,5)	A_{320}	(1,2), (2,3), (2,8), (3,4), (4,5), (5,2), (5,6), (6,7), (7,5), (8,1), (8,7)
A_{323}	(1,2), (2,3), (3,1), (3,4), (3,6), (4,2), (4,5), (5,3), (6,7), (7,8), (8,5)	A_{325}	(1,2), (2,3), (2,7), (3,4), (4,2), (4,5), (5,6), (6,7), (7,4), (7,8), (8,2)
A_{331}	(1,2), (2,3), (2,4), (2,8), (3,1), (4,1), (4,5), (5,6), (6,2), (6,7), (7,5), (8,6)	A_{339}	(1,2), (1,6), (2,3), (3,1), (3,4), (4,5), (4,8), (5,6), (5,7), (6,3), (7,4), (8,3)
A_{341}	(1,2), (2,3), (2,8), (3,1), (3,4), (4,2), (4,5), (4,6), (5,3), (6,7), (7,8), (8,4)	A_{358}	(1,2), (2,3), (3,4), (3,7), (4,1), (4,5), (5,3), (5,6), (6,7), (7,5), (7,8), (8,3)
A_{376}	(1,2), (2,3), (3,4), (3,5), (3,8), (4,2), (5,6), (5,7), (6,3), (7,3), (8,1), (8,7)	A_{377}	(1,2), (2,3), (2,6), (2,8), (3,1), (3,4), (4,5), (5,2), (6,5), (6,7), (7,2), (8,7)
A_{384}	(1,2), (2,3), (2,6), (3,1), (3,4), (4,2), (4,5), (5,6), (5,8), (6,4), (6,7), (7,5), (8,4)	A_{385}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (4,8), (5,3), (5,6), (6,4), (6,7), (7,5), (8,6)
A_{387}	(1,2), (2,3), (2,7), (3,1), (3,4), (4,2), (4,5), (4,6), (5,3), (6,7), (6,8), (7,4), (8,4)	A_{391}	(1,2), (2,3), (2,8), (3,1), (3,4), (4,2), (4,5), (4,7), (5,3), (5,6), (6,4), (7,6), (8,4)

$\boldsymbol{6(x^8+x^7+2x^4+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{38}	(1,2), (2,3), (3,1), (3,4), (4,5), (5,6), (6,7), (7,4), (7,8)	A_{39}	(1,2), (2,3), (3,4), (3,5), (4,2), (5,6), (6,7), (7,3), (7,8)
A_{45}	(1,2), (2,3), (3,4), (4,5), (5,2), (5,6), (6,7), (7,8), (8,6)	A_{50}	(2,1), (2,3), (3,4), (3,6), (4,5), (5,2), (6,7), (7,3), (8,6)
A_{56}	(2,1), (2,3), (3,4), (4,5), (5,2), (6,5), (6,7), (7,8), (8,6)	A_{57}	(1,2), (2,3), (3,4), (3,5), (4,2), (5,6), (5,8), (6,7), (7,3)
A_{62}	(1,2), (2,3), (3,4), (3,7), (4,5), (5,2), (5,6), (7,8), (8,3)	A_{68}	(2,1), (2,3), (3,4), (3,6), (4,5), (5,2), (6,7), (7,8), (8,6)
A_{73}	(2,1), (2,3), (3,4), (4,5), (5,2), (5,6), (6,7), (7,5), (8,6)	A_{75}	(2,1), (2,3), (3,4), (4,5), (5,2), (5,6), (6,7), (7,5), (8,3)
A_{97}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,2), (5,6), (6,7), (6,8), (7,4)	A_{108}	(1,2), (2,3), (3,1), (4,2), (4,5), (5,6), (6,7), (7,4), (7,8), (8,6)
A_{114}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,6), (6,7), (6,8), (7,5), (8,4)	A_{115}	(1,2), (2,3), (3,1), (3,4), (4,5), (5,3), (5,6), (6,7), (7,4), (8,7)
A_{117}	(1,2), (2,3), (3,4), (3,5), (4,2), (5,6), (6,7), (6,8), (7,3), (8,5)	A_{138}	(1,2), (2,3), (2,5), (3,4), (4,1), (5,1), (6,2), (6,7), (7,8), (8,6)
A_{139}	(1,2), (2,3), (3,4), (3,5), (3,8), (4,2), (5,6), (6,7), (7,3), (8,7)	A_{141}	(1,2), (2,3), (2,4), (4,5), (4,6), (5,1), (6,2), (6,7), (7,8), (8,6)
A_{145}	(1,2), (2,3), (3,4), (3,5), (4,2), (5,1), (5,6), (6,7), (7,5), (7,8)	A_{147}	(1,2), (2,3), (3,4), (4,1), (4,5), (4,6), (5,3), (6,7), (7,8), (8,6)
A_{156}	(1,2), (2,3), (2,4), (3,1), (4,5), (4,6), (5,2), (6,7), (6,8), (7,2)	A_{157}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,6), (6,7), (7,4), (7,8), (8,6)

algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{158}	(1,2), (2,3), (3,4), (4,2), (4,5), (5,6), (6,7), (6,8), (7,4), (8,5)	A_{164}	(1,2), (2,3), (3,4), (3,5), (3,6), (4,1), (6,2), (6,7), (7,8), (8,6)
A_{167}	(1,2), (2,3), (3,4), (4,5), (4,6), (4,8), (5,2), (6,7), (7,4), (8,3)	A_{180}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,8), (5,6), (6,7), (7,4), (8,7)
A_{188}	(2,1), (2,3), (3,4), (4,2), (4,5), (5,6), (6,7), (7,4), (7,8), (8,6)	A_{189}	(1,2), (2,3), (3,1), (4,3), (4,5), (5,6), (5,7), (6,4), (7,8), (8,4)
A_{190}	(1,2), (2,3), (3,1), (4,3), (4,5), (5,6), (6,7), (6,8), (7,4), (8,5)	A_{191}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,6), (6,2), (6,7), (7,5), (8,5)
A_{194}	(2,1), (2,3), (2,6), (3,4), (4,5), (4,7), (5,2), (6,5), (7,8), (8,4)	A_{195}	(2,1), (2,3), (3,4), (4,2), (4,5), (5,6), (5,8), (6,7), (7,4), (8,4)
A_{197}	(2,1), (2,3), (3,4), (3,5), (4,2), (5,6), (5,8), (6,7), (7,3), (8,3)	A_{198}	(1,2), (2,3), (3,1), (3,4), (4,5), (5,6), (5,8), (6,7), (7,4), (8,4)
A_{210}	(2,1), (2,3), (3,4), (3,5), (3,7), (4,2), (5,6), (6,2), (7,8), (8,3)	A_{217}	(1,2), (2,3), (2,8), (3,4), (4,5), (5,2), (5,6), (6,7), (7,5), (8,5)
A_{228}	(1,2), (2,3), (2,4), (3,1), (4,1), (4,5), (5,2), (6,4), (6,7), (7,8), (8,6)	A_{229}	(1,2), (2,3), (3,4), (3,5), (3,8), (4,2), (5,6), (6,3), (6,7), (7,5), (8,6)
A_{230}	(1,2), (2,3), (2,6), (3,4), (4,2), (4,5), (5,6), (5,7), (6,4), (7,8), (8,5)	A_{233}	(2,1), (2,3), (3,4), (3,5), (4,2), (5,2), (5,6), (6,3), (6,7), (7,8), (8,6)
A_{235}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,6), (6,3), (6,7), (7,4), (7,8), (8,6)	A_{245}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,8), (5,6), (6,4), (6,7), (7,5), (8,6)
A_{248}	(1,2), (2,3), (3,1), (4,2), (4,5), (5,6), (5,7), (6,4), (7,4), (7,8), (8,5)	A_{251}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (5,3), (5,6), (6,7), (7,5), (8,4)
A_{253}	(1,2), (2,3), (2,4), (3,1), (4,1), (4,5), (4,6), (5,2), (6,7), (7,8), (8,6)	A_{254}	(1,2), (2,3), (3,4), (3,5), (4,2), (5,6), (5,7), (6,3), (7,3), (7,8), (8,5)
A_{255}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,6), (5,8), (6,4), (6,7), (7,5), (8,4)	A_{257}	(1,2), (2,3), (3,4), (3,5), (3,7), (4,8), (5,6), (5,7), (6,4), (7,4), (8,7)
A_{257}	(1,2), (2,3), (3,4), (3,5), (3,7), (4,2), (5,6), (6,3), (7,6), (7,8), (8,3)	A_{264}	(2,1), (2,3), (3,4), (4,2), (4,5), (5,6), (5,7), (6,4), (7,4), (7,8), (8,5)
A_{284}	(2,1), (2,3), (2,5), (3,4), (4,2), (5,4), (5,6), (6,2), (6,7), (7,8), (8,6)	A_{287}	(1,2), (2,3), (3,1), (4,3), (4,5), (4,7), (5,6), (5,8), (6,4), (7,8), (8,4)
A_{289}	(2,1), (2,3), (3,4), (3,6), (3,8), (4,2), (4,5), (5,3), (6,7), (7,3), (8,2)	A_{291}	(1,2), (2,3), (3,1), (3,4), (3,5), (5,2), (5,6), (5,7), (6,3), (7,8), (8,5)
A_{292}	(1,2), (2,3), (3,4), (4,2), (4,5), (4,7), (5,6), (6,4), (7,6), (7,8), (8,4)	A_{294}	(1,2), (2,3), (2,5), (2,6), (3,1), (3,4), (4,2), (6,4), (6,7), (7,8), (8,6)
A_{298}	(1,2), (2,3), (2,6), (3,4), (4,2), (4,5), (4,7), (5,6), (6,4), (7,8), (8,4)	A_{316}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,6), (5,3), (6,3), (6,7), (7,4), (8,4)
A_{336}	(1,2), (2,3), (3,1), (3,4), (4,5), (5,3), (5,6), (6,4), (6,7), (7,5), (7,8), (8,6)	A_{344}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (5,3), (5,6), (5,7), (6,4), (7,8), (8,5)
A_{347}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,8), (5,3), (5,6), (6,4), (6,7), (7,8), (8,6)	A_{359}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,6), (5,3), (6,3), (6,7), (7,4), (7,8), (8,6)

$\boldsymbol{8(x^8+2x^7+2x^4+2x+1)}$	
algebra KQ/I	quiver Q
A_{91}	(1,2), (2,3), (2,5), (3,1), (4,2), (5,6), (6,2), (6,7), (7,8), (8,6)
A_{101}	(1,2), (2,3), (2,5), (3,1), (4,2), (5,6), (5,7), (6,2), (7,8), (8,5)

$\boldsymbol{8(x^8+x^7+x^6+2x^4+x^2+x+1)}$			
algebra KQ/I	quiver Q	algebra KQ/I	quiver Q
A_{59}	(1,2), (2,3), (3,4), (3,6), (4,5), (5,1), (6,7), (7,8), (8,6)	A_{63}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,6), (6,7), (7,8), (8,4)
A_{64}	(1,2), (2,3), (3,4), (3,5), (4,2), (5,6), (6,7), (7,8), (8,3)	A_{79}	(1,2), (2,3), (3,4), (4,5), (5,6), (5,7), (6,2), (7,8), (8,5)
A_{81}	(2,1), (2,3), (3,4), (4,5), (4,7), (5,6), (6,2), (7,8), (8,4)	A_{82}	(1,2), (2,3), (3,4), (4,2), (4,5), (5,6), (6,7), (7,8), (8,4)
A_{130}	(1,2), (2,3), (3,1), (3,4), (4,5), (5,6), (6,3), (6,7), (7,8), (8,5)	A_{168}	(1,2), (2,3), (3,4), (4,1), (4,5), (5,6), (6,3), (6,7), (7,8), (8,6)
A_{236}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,2), (5,6), (6,4), (6,7), (7,8), (8,5)	A_{244}	(1,2), (2,3), (3,4), (4,1), (4,5), (5,3), (5,6), (5,7), (6,4), (7,8), (8,5)
A_{250}	(1,2), (2,3), (3,1), (3,4), (4,5), (5,6), (5,7), (6,3), (7,4), (7,8), (8,5)	A_{259}	(1,2), (2,3), (2,4), (3,1), (4,5), (5,6), (5,8), (6,2), (6,7), (7,5), (8,7)
A_{269}	(1,2), (1,6), (2,3), (3,1), (3,4), (4,5), (4,7), (5,6), (6,3), (7,8), (8,4)	A_{271}	(1,2), (2,3), (3,4), (3,6), (4,1), (4,5), (5,3), (5,7), (6,5), (7,8), (8,5)
A_{288}	(1,2), (2,3), (3,1), (3,4), (4,5), (4,7), (5,3), (5,6), (6,4), (7,8), (8,6)	A_{301}	(1,2), (2,3), (3,1), (3,4), (4,2), (4,5), (5,6), (6,3), (6,7), (7,8), (8,6)
A_{329}	(1,2), (2,3), (3,1), (3,4), (3,6), (4,2), (4,5), (5,3), (6,5), (6,7), (7,8), (8,6)	A_{330}	(1,2), (2,3), (3,1), (3,4), (3,6), (4,5), (5,3), (5,7), (6,5), (7,6), (7,8), (8,5)
A_{332}	(1,2), (2,3), (2,4), (2,8), (3,1), (4,1), (4,5), (5,2), (5,6), (6,7), (7,5), (8,5)	A_{334}	(1,2), (2,3), (2,5), (3,1), (3,4), (4,2), (5,1), (5,6), (6,2), (6,7), (7,8), (8,6)

D Derived equivalences for cluster-tilted algebras of type E_8

D.1 Polynomial $2(x^8 - x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 + 1)$

$A_2^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_2$, $A_{19}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{28}$

A_2	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{19}	$(56)(78)$	$\Rightarrow$	$A_2 \underset{\text{der}}{\sim} A_{28}$

D.2 Polynomial $2(x^8 - x^6 + x^5 + x^3 - x^2 + 1)$

$A_3^{\text{op}} \underset{\text{s/s}}{\sim} A_{10}$, $A_4^{\text{op}} \underset{\text{s/s}}{\sim} A_7$, $A_5^{\text{op}} \underset{\text{s/s}}{\sim} A_5$, $A_6^{\text{op}} \underset{\text{s/s}}{\sim} A_6$, $A_{23}^{\text{op}} \underset{\text{s/s}}{\sim} A_{35}$, $A_{25}^{\text{op}} \underset{\text{s/s}}{\sim} A_{25}$, $A_{31}^{\text{op}} \underset{\text{s/s}}{\sim} A_{46}$

A_3	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_6	(17264358)	$\Rightarrow$	$A_{10} \underset{\text{der}}{\sim} A_6$
A_3	$(4;3,6,7)$	$\underset{\text{der}}{\sim}$	A_{23}	(56)	$\Rightarrow$	$A_{10} \underset{\text{der}}{\sim} A_{35}$
A_6	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{31}	$(17)(246)$	$\Rightarrow$	$A_6 \underset{\text{der}}{\sim} A_{46}$
A_7^{op}	$(5;3,6)$	$\underset{\text{der}}{\sim}$	A_3	$(18)(27)(3465)$	$\Rightarrow$	$A_7 \underset{\text{der}}{\sim} A_{10}$
A_5^{op}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_4	(678)	$\Rightarrow$	$A_5 \underset{\text{der}}{\sim} A_4$

D.3 Polynomial $3(x^8 + x^4 + 1)$

$A_8^{\text{op}} \underset{\text{s/s}}{\sim} A_9$, $A_{12}^{\text{op}} \underset{\text{s/s}}{\sim} A_{14}$, $A_{17}^{\text{op}} = A_{17}$, $A_{26}^{\text{op}} \underset{\text{s/s}}{\sim} A_{34}$, $A_{30}^{\text{op}} \underset{\text{s/s}}{\sim} A_{33}$, $A_{43}^{\text{op}} \underset{\text{s/s}}{\sim} A_{60}$, $A_{44}^{\text{op}} \underset{\text{s/s}}{\sim} A_{66}$, $A_{47}^{\text{op}} \underset{\text{s/s}}{\sim} A_{67}$, $A_{53}^{\text{op}} = A_{61}$, $A_{76}^{\text{op}} \underset{\text{s/s}}{\sim} A_{80}$, $A_{84}^{\text{op}} \underset{\text{s/s}}{\sim} A_{84}$, $A_{92}^{\text{op}} \underset{\text{s/s}}{\sim} A_{109}$, $A_{94}^{\text{op}} \underset{\text{s/s}}{\sim} A_{100}$, $A_{102}^{\text{op}} = A_{148}$, $A_{110}^{\text{op}} = A_{131}$, $A_{111}^{\text{op}} = A_{171}$, $A_{123}^{\text{op}} \underset{\text{s/s}}{\sim} A_{123}$, $A_{132}^{\text{op}} = A_{149}$, $A_{144}^{\text{op}} \underset{\text{s/s}}{\sim} A_{187}$, $A_{154}^{\text{op}} \underset{\text{s/s}}{\sim} A_{163}$, $A_{169}^{\text{op}} = A_{196}$, $A_{173}^{\text{op}} \underset{\text{s/s}}{\sim} A_{173}$, $A_{206}^{\text{op}} \underset{\text{s/s}}{\sim} A_{218}$, $A_{221}^{\text{op}} = A_{222}$, $A_{232}^{\text{op}} \underset{\text{s/s}}{\sim} A_{242}$, $A_{247}^{\text{op}} \underset{\text{s/s}}{\sim} A_{277}$, $A_{272}^{\text{op}} = A_{275}$, $A_{305}^{\text{op}} = A_{305}$

A_8	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{100}	$(45)(678)$	$\Rightarrow$	$A_9 \underset{\text{der}}{\sim} A_{94}$
A_9	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{109}	(576)	$\Rightarrow$	$A_8 \underset{\text{der}}{\sim} A_{92}$
A_8	$(4;3,7),(6;5)$	$\underset{\text{der}}{\sim}$	A_{26}	$(45)(67)$	$\Rightarrow$	$A_9 \underset{\text{der}}{\sim} A_{34}$
A_9	$(5;4,8),(7;6)$	$\underset{\text{der}}{\sim}$	A_{30}	(57)	$\Rightarrow$	$A_8 \underset{\text{der}}{\sim} A_{33}$
A_{12}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{154}	(465)	$\Rightarrow$	$A_{14} \underset{\text{der}}{\sim} A_{163}$
A_{14}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{196}	$(18)(275)(46)$	$\Rightarrow$	$A_{12} \underset{\text{der}}{\sim} A_{169}$
A_{12}	$(4;3,7),(6;5)$	$\underset{\text{der}}{\sim}$	A_{47}	(46)	$\Rightarrow$	$A_{14} \underset{\text{der}}{\sim} A_{67}$
A_{14}	$(4;3),(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{61}	(4756)	$\Rightarrow$	$A_{12} \underset{\text{der}}{\sim} A_{53}$
A_{17}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{218}	(185236)	$\Rightarrow$	$A_{17} \underset{\text{der}}{\sim} A_{206}$
$A_{218}(*)$	$(4;3,5,7)$	$\underset{\text{der}}{\sim}$	A_{196}	(37654)	$\Rightarrow$	$A_{206} \underset{\text{der}}{\sim} A_{169}$
$A_{196}(*)$	$(2;1,5,7)$	$\underset{\text{der}}{\sim}$	A_{80}	$(178)(243)(56)$	$\Rightarrow$	$A_{169} \underset{\text{der}}{\sim} A_{76}$
$A_{44}(*)$	$(4;3,7,8)$	$\underset{\text{der}}{\sim}$	A_{92}	(567)	$\Rightarrow$	$A_{66} \underset{\text{der}}{\sim} A_{109}$
$A_{102}(*)$	$(4;3,6,7)$	$\underset{\text{der}}{\sim}$	A_{43}	(567)	$\Rightarrow$	$A_{148} \underset{\text{der}}{\sim} A_{60}$
A_{102}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{132}	(35674)	$\Rightarrow$	$A_{148} \underset{\text{der}}{\sim} A_{149}$
$A_{132}(*)$	$(3;2,4,7)$	$\underset{\text{der}}{\sim}$	A_{247}	(1)	$\Rightarrow$	$A_{149} \underset{\text{der}}{\sim} A_{277}$
$A_{275}(*)$	$(7;3,6,8)$	$\underset{\text{der}}{\sim}$	A_{111}	$(183546)(27)$	$\Rightarrow$	$A_{272} \underset{\text{der}}{\sim} A_{171}$
$A_{154}(*)$	$(3;2,5,6)$	$\underset{\text{der}}{\sim}$	A_{84}	$(34)(576)$	$\Rightarrow$	$A_{163} \underset{\text{der}}{\sim} A_{84}$
$A_{173}(*)$	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{218}	(34657)	$\Rightarrow$	$A_{173} \underset{\text{der}}{\sim} A_{206}$
$A_{187}(*)$	$(5;4,6,8)$	$\underset{\text{der}}{\sim}$	A_{154}	(485)	$\Rightarrow$	$A_{144} \underset{\text{der}}{\sim} A_{163}$
A_{123}	$(4;3,6,8)$	$\underset{\text{der}}{\sim}$	A_{163}	(4587)	$\Rightarrow$	$A_{123} \underset{\text{der}}{\sim} A_{154}$
$A_{131}(*)$	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{53}	(4756)	$\Rightarrow$	$A_{110} \underset{\text{der}}{\sim} A_{61}$
A_{305}	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{222}	(1876423)	$\Rightarrow$	$A_{305} \underset{\text{der}}{\sim} A_{221}$
$A_{44}(*)$	$(4;3,7,8),(6;5)$	$\underset{\text{der}}{\sim}$	A_{43}	(568)	$\Rightarrow$	$A_{66} \underset{\text{der}}{\sim} A_{60}$
A_{232}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{44}	(48675)	$\Rightarrow$	$A_{242} \underset{\text{der}}{\sim} A_{66}$
A_{272}	$(4;3,6)$	$\underset{\text{der}}{\sim}$	A_{84}	(387564)	$\Rightarrow$	$A_{275} \underset{\text{der}}{\sim} A_{84}$
A_{305}	$(6;2,5)$	$\underset{\text{der}}{\sim}$	A_{80}	$(18)(24635)$	$\Rightarrow$	$A_{305} \underset{\text{der}}{\sim} A_{76}$
$A_{132}(*)$	$(3;2,4,7)$	$\underset{\text{der}}{\sim}$	A_{53}	$(47)(56)$	$\Rightarrow$	$A_{149} \underset{\text{der}}{\sim} A_{61}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.4 Polynomial $4(x^8 + x^7 - x^6 + x^5 + x^3 - x^2 + x + 1)$

$A_{20}^{\text{op}} \underset{\text{s/s}}{\sim} A_{41}$, $A_{21}^{\text{op}} = A_{49}$, $A_{22}^{\text{op}} \underset{\text{s/s}}{\sim} A_{52}$, $A_{27}^{\text{op}} \underset{\text{s/s}}{\sim} A_{27}$, $A_{29}^{\text{op}} = A_{36}$, $A_{37}^{\text{op}} \underset{\text{s/s}}{\sim} A_{37}$, $A_{89}^{\text{op}} = A_{122}$, $A_{90}^{\text{op}} \underset{\text{s/s}}{\sim} A_{142}$, $A_{98}^{\text{op}} \underset{\text{s/s}}{\sim} A_{106}$, $A_{105}^{\text{op}} \underset{\text{s/s}}{\sim} A_{124}$

A_{20}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{27}	$(18765)(23)$	$\Rightarrow$	$A_{41} \underset{\text{der}}{\sim} A_{27}$
A_{21}	$(2;1,4,5)$	$\underset{\text{der}}{\sim}$	A_{90}	$(18)(2647)(35)$	$\Rightarrow$	$A_{49} \underset{\text{der}}{\sim} A_{142}$
A_{49}	$(5;2,7)$	$\underset{\text{der}}{\sim}$	A_{27}	$(178)(24536)$	$\Rightarrow$	$A_{21} \underset{\text{der}}{\sim} A_{27}$
A_{22}	$(2;1,4,5)$	$\underset{\text{der}}{\sim}$	A_{89}	(34)	$\Rightarrow$	$A_{52} \underset{\text{der}}{\sim} A_{122}$
A_{52}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{36}	(34)	$\Rightarrow$	$A_{22} \underset{\text{der}}{\sim} A_{29}$
A_{36}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{20}	(45)	$\Rightarrow$	$A_{29} \underset{\text{der}}{\sim} A_{41}$
A_{37}^{op}	$(6;5,7)$	$\underset{\text{der}}{\sim}$	A_{21}	$(23)(678)$	$\Rightarrow$	$A_{37} \underset{\text{der}}{\sim} A_{49}$
A_{37}^{op}	$(2;3,4,5)$	$\underset{\text{der}}{\sim}$	A_{124}	$(143)(78)$	$\Rightarrow$	$A_{37} \underset{\text{der}}{\sim} A_{105}$
A_{122}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{106}	(67)	$\Rightarrow$	$A_{89} \underset{\text{der}}{\sim} A_{98}$

D.5 Polynomial $4(x^8 + x^7 - x^6 + 2x^4 - x^2 + x + 1)$

$A_{24}^{\text{op}} \underset{\text{s/s}}{\sim} A_{32}$, $A_{93}^{\text{op}} = A_{121}$, $A_{107}^{\text{op}} \underset{\text{s/s}}{\sim} A_{153}$, $A_{113}^{\text{op}} \underset{\text{s/s}}{\sim} A_{152}$, $A_{120}^{\text{op}} = A_{146}$, $A_{137}^{\text{op}} = A_{155}$

A_{32}	$(5;3,8)$	$\underset{\text{der}}{\sim}$	A_{24}	(5687)		
A_{32}	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{113}	$(18267)(345)$	$\Rightarrow$	$A_{24} \underset{\text{der}}{\sim} A_{152}$
A_{113}	$(3;2,5,6)$	$\underset{\text{der}}{\sim}$	A_{107}	$(13478)(26)$	$\Rightarrow$	$A_{152} \underset{\text{der}}{\sim} A_{153}$
A_{155}	$(6;3,8)$	$\underset{\text{der}}{\sim}$	A_{120}	$(18)(27456)$	$\Rightarrow$	$A_{137} \underset{\text{der}}{\sim} A_{146}$
A_{93}	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{107}	(67)	$\Rightarrow$	$A_{121} \underset{\text{der}}{\sim} A_{153}$
A_{120}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{153}	$(18)(26547)$	$\Rightarrow$	$A_{146} \underset{\text{der}}{\sim} A_{107}$

D.6 Polynomial $4(x^8 + x^7 - 2x^6 + 2x^5 + 2x^3 - 2x^2 + x + 1)$

$A_{95}^{\text{op}} = A_{119}$, $A_{96}^{\text{op}} \underset{\text{s/s}}{\sim} A_{116}$

A_{95}^{op}	$(3;1,4)$	$\underset{\text{der}}{\sim}$	A_{96}	$(243)(56)$	$\Rightarrow$	$A_{95} \underset{\text{der}}{\sim} A_{116}$
A_{96}	$(2;1),(4;3)$	$\underset{\text{der}}{\sim}$	A_{95}	$(12)(34)$	$\Rightarrow$	$A_{116} \underset{\text{der}}{\sim} A_{119}$

D.7 Polynomial $4(x^8 + x^6 - x^5 + 2x^4 - x^3 + x^2 + 1)$

$A_{11}^{\text{op}} \underset{\text{s/s}}{\sim} A_{11}$, $A_{13}^{\text{op}} \underset{\text{s/s}}{\sim} A_{16}$, $A_{40}^{\text{op}} \underset{\text{s/s}}{\sim} A_{40}$, $A_{42}^{\text{op}} \underset{\text{s/s}}{\sim} A_{55}$, $A_{54}^{\text{op}} = A_{54}$, $A_{58}^{\text{op}} \underset{\text{s/s}}{\sim} A_{72}$, $A_{85}^{\text{op}} = A_{87}$, $A_{99}^{\text{op}} \underset{\text{s/s}}{\sim} A_{128}$, $A_{103}^{\text{op}} \underset{\text{s/s}}{\sim} A_{162}$, $A_{104}^{\text{op}} \underset{\text{s/s}}{\sim} A_{136}$, $A_{112}^{\text{op}} \underset{\text{s/s}}{\sim} A_{129}$, $A_{126}^{\text{op}} \underset{\text{s/s}}{\sim} A_{143}$, $A_{127}^{\text{op}} \underset{\text{s/s}}{\sim} A_{185}$, $A_{133}^{\text{op}} = A_{150}$, $A_{134}^{\text{op}} \underset{\text{s/s}}{\sim} A_{134}$, $A_{135}^{\text{op}} \underset{\text{s/s}}{\sim} A_{182}$, $A_{151}^{\text{op}} = A_{172}$, $A_{170}^{\text{op}} = A_{175}$, $A_{176}^{\text{op}} = A_{177}$, $A_{186}^{\text{op}} \underset{\text{s/s}}{\sim} A_{193}$, $A_{192}^{\text{op}} \underset{\text{s/s}}{\sim} A_{207}$, $A_{208}^{\text{op}} \underset{\text{s/s}}{\sim} A_{208}$, $A_{224}^{\text{op}} \underset{\text{s/s}}{\sim} A_{231}$, $A_{225}^{\text{op}} = A_{239}$, $A_{226}^{\text{op}} = A_{237}$, $A_{227}^{\text{op}} \underset{\text{s/s}}{\sim} A_{258}$, $A_{238}^{\text{op}} = A_{240}$, $A_{243}^{\text{op}} \underset{\text{s/s}}{\sim} A_{286}$, $A_{273}^{\text{op}} = A_{276}$, $A_{282}^{\text{op}} = A_{304}$, $A_{311}^{\text{op}} = A_{324}$, $A_{333}^{\text{op}} = A_{336}$, $A_{337}^{\text{op}} \underset{\text{s/s}}{\sim} A_{338}$, $A_{342}^{\text{op}} \underset{\text{s/s}}{\sim} A_{343}$, $A_{352}^{\text{op}} = A_{370}$, $A_{361}^{\text{op}} = A_{366}$, $A_{388}^{\text{op}} = A_{388}$

A_{11}	$(4;3,8)$	$\underset{\text{der}}{\sim}$	A_{162}	$(45)(678)$	$\Rightarrow$	$A_{11} \underset{\text{der}}{\sim} A_{103}$
A_{11}	$(4;3,8),(6;5)$	$\underset{\text{der}}{\sim}$	A_{231}	$(45)(68)$	$\Rightarrow$	$A_{11} \underset{\text{der}}{\sim} A_{224}$
A_{13}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{192}	$(34)(567)$	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_{207}$
A_{13}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{112}	(576)	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_{129}$
A_{13}	$(5;4,8),(7;6)$	$\underset{\text{der}}{\sim}$	A_{225}	$(56)(78)$	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_{239}$
A_{13}	$(3;2,7),(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{333}	$(34)(57)$	$\Rightarrow$	$A_{16} \underset{\text{der}}{\sim} A_{363}$
A_{40}^{op}	$(2;1,3)$	$\underset{\text{der}}{\sim}$	A_{258}	$(18)(27)(35)$	$\Rightarrow$	$A_{40} \underset{\text{der}}{\sim} A_{227}$
A_{40}^{op}	$(2;1,3),(5;6,7)$	$\underset{\text{der}}{\sim}$	A_{231}	$(18)(27)(3645)$	$\Rightarrow$	$A_{40} \underset{\text{der}}{\sim} A_{224}$
A_{42}	$(3;2,6)$	$\underset{\text{der}}{\sim}$	A_{243}	$(1827)(3645)$	$\Rightarrow$	$A_{55} \underset{\text{der}}{\sim} A_{286}$
A_{42}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{224}	$(56)(78)$	$\Rightarrow$	$A_{55} \underset{\text{der}}{\sim} A_{231}$
A_{42}^{op}	$(6;3,7)$	$\underset{\text{der}}{\sim}$	A_{239}	(48765)	$\Rightarrow$	$A_{42} \underset{\text{der}}{\sim} A_{225}$
A_{54}	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{276}	(587)	$\Rightarrow$	$A_{54} \underset{\text{der}}{\sim} A_{273}$
A_{54}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{226}	$(354)(67)$	$\Rightarrow$	$A_{54} \underset{\text{der}}{\sim} A_{237}$
A_{54}	$(3;2,7),(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{240}	$(354)(67)$	$\Rightarrow$	$A_{54} \underset{\text{der}}{\sim} A_{238}$
$A_{58}(*)$	$(3;2,7,8)$	$\underset{\text{der}}{\sim}$	A_{104}	(4567)	$\Rightarrow$	$A_{72} \underset{\text{der}}{\sim} A_{136}$
A_{58}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{143}	(458)	$\Rightarrow$	$A_{72} \underset{\text{der}}{\sim} A_{126}$
A_{104}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{129}	(468)	$\Rightarrow$	$A_{136} \underset{\text{der}}{\sim} A_{112}$

$A_{85}(*)$	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{176}	$(56)(78)$	$\Rightarrow$	$A_{87} \underset{\text{der}}{\sim} A_{177}$
A_{87}	$(3;2)$	$\underset{\text{der}}{\sim}$	A_{208}	$(18)(246375)$	$\Rightarrow$	$A_{85} \underset{\text{der}}{\sim} A_{208}$
A_{176}	$(4;3,6)$	$\underset{\text{der}}{\sim}$	A_{133}	(45)	$\Rightarrow$	$A_{177} \underset{\text{der}}{\sim} A_{150}$
A_{208}	$(4;3,7,8)$	$\underset{\text{der}}{\sim}$	A_{342}	(4576)	$\Rightarrow$	$A_{208} \underset{\text{der}}{\sim} A_{343}$
A_{99}^{op}	$(5;6,7,8)$	$\underset{\text{der}}{\sim}$	A_{237}	$(17)(2635)$	$\Rightarrow$	$A_{99} \underset{\text{der}}{\sim} A_{226}$
A_{127}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{126}	(15362478)	$\Rightarrow$	$A_{185} \underset{\text{der}}{\sim} A_{143}$
A_{127}^{op}	$(2;1,3,8)$	$\underset{\text{der}}{\sim}$	A_{238}	$(18)(46)$	$\Rightarrow$	$A_{127} \underset{\text{der}}{\sim} A_{240}$
A_{134}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{342}	(5876)	$\Rightarrow$	$A_{134} \underset{\text{der}}{\sim} A_{343}$
A_{135}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{103}	(56)	$\Rightarrow$	$A_{182} \underset{\text{der}}{\sim} A_{162}$
A_{135}	$(3,2,6)$	$\underset{\text{der}}{\sim}$	A_{343}	$(1845)(2736)$	$\Rightarrow$	$A_{182} \underset{\text{der}}{\sim} A_{342}$
$A_{172}(*)$	$(2;1,4,7)$	$\underset{\text{der}}{\sim}$	A_{282}	(134)	$\Rightarrow$	$A_{151} \underset{\text{der}}{\sim} A_{304}$
A_{282}	$(7;2,6)$	$\underset{\text{der}}{\sim}$	A_{311}	$(38)(4657)$	$\Rightarrow$	$A_{304} \underset{\text{der}}{\sim} A_{324}$
$A_{324}(*)$	$(2;1,4,6)$	$\underset{\text{der}}{\sim}$	A_{366}	$(134)(687)$	$\Rightarrow$	$A_{311} \underset{\text{der}}{\sim} A_{361}$
A_{170}	$(3;2,7)$	$\underset{\text{der}}{\sim}$	A_{361}	$(34)(5867)$	$\Rightarrow$	$A_{175} \underset{\text{der}}{\sim} A_{366}$
A_{170}	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{150}	$(18)(274356)$	$\Rightarrow$	$A_{175} \underset{\text{der}}{\sim} A_{133}$
$A_{186}(*)$	$(3;2,6,8)$	$\underset{\text{der}}{\sim}$	A_{363}	$(1625)(34)$	$\Rightarrow$	$A_{193} \underset{\text{der}}{\sim} A_{333}$
A_{337}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{352}	(23)	$\Rightarrow$	$A_{338} \underset{\text{der}}{\sim} A_{370}$
A_{352}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{333}	(345)	$\Rightarrow$	$A_{370} \underset{\text{der}}{\sim} A_{363}$
A_{388}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{324}	(3465)	$\Rightarrow$	$A_{388} \underset{\text{der}}{\sim} A_{311}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.8 Polynomial $4(x^8 + x^6 - 2x^5 + 4x^4 - 2x^3 + x^2 + 1)$

$A_{48}^{\text{op}} \underset{\text{s/s}}{\sim} A_{70}$, $A_{118}^{\text{op}} \underset{\text{s/s}}{\sim} A_{160}$, $A_{161}^{\text{op}} \underset{\text{s/s}}{\sim} A_{161}$, $A_{278}^{\text{op}} = A_{302}$

A_{161}	$(2;1,6)$	$\underset{\text{der}}{\sim}$	A_{160}	(3456)	$\Rightarrow$	$A_{161} \underset{\text{der}}{\sim} A_{118}$
A_{48}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{118}	(56)	$\Rightarrow$	$A_{70} \underset{\text{der}}{\sim} A_{160}$
$A_{70}(*)$	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{302}	(576)	$\Rightarrow$	$A_{48} \underset{\text{der}}{\sim} A_{278}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.9 Polynomial $5(x^8 + x^6 + x^4 + x^2 + 1)$

$A_{18}^{\text{op}} = A_{18}$, $A_{51}^{\text{op}} \underset{\text{s/s}}{\sim} A_{69}$, $A_{65}^{\text{op}} = A_{71}$, $A_{74}^{\text{op}} \underset{\text{s/s}}{\sim} A_{78}$, $A_{77}^{\text{op}} = A_{86}$, $A_{83}^{\text{op}} \underset{\text{s/s}}{\sim} A_{83}$, $A_{125}^{\text{op}} = A_{159}$, $A_{140}^{\text{op}} = A_{174}$, $A_{165}^{\text{op}} \underset{\text{s/s}}{\sim} A_{166}$, $A_{178}^{\text{op}} = A_{204}$, $A_{181}^{\text{op}} = A_{202}$, $A_{183}^{\text{op}} \underset{\text{s/s}}{\sim} A_{200}$, $A_{199}^{\text{op}} \underset{\text{s/s}}{\sim} A_{203}$, $A_{201}^{\text{op}} \underset{\text{s/s}}{\sim} A_{201}$, $A_{212}^{\text{op}} \underset{\text{s/s}}{\sim} A_{220}$, $A_{213}^{\text{op}} \underset{\text{s/s}}{\sim} A_{219}$, $A_{214}^{\text{op}} \underset{\text{s/s}}{\sim} A_{216}$, $A_{223}^{\text{op}} = A_{223}$, $A_{234}^{\text{op}} \underset{\text{s/s}}{\sim} A_{234}$, $A_{241}^{\text{op}} = A_{274}$, $A_{246}^{\text{op}} = A_{265}$, $A_{249}^{\text{op}} = A_{260}$, $A_{252}^{\text{op}} \underset{\text{s/s}}{\sim} A_{266}$, $A_{261}^{\text{op}} \underset{\text{s/s}}{\sim} A_{285}$, $A_{262}^{\text{op}} \underset{\text{s/s}}{\sim} A_{295}$, $A_{263}^{\text{op}} \underset{\text{s/s}}{\sim} A_{293}$, $A_{267}^{\text{op}} = A_{281}$, $A_{279}^{\text{op}} = A_{303}$, $A_{283}^{\text{op}} \underset{\text{s/s}}{\sim} A_{296}$, $A_{297}^{\text{op}} = A_{310}$, $A_{306}^{\text{op}} = A_{307}$, $A_{312}^{\text{op}} \underset{\text{s/s}}{\sim} A_{314}$, $A_{315}^{\text{op}} = A_{318}$, $A_{321}^{\text{op}} = A_{327}$, $A_{322}^{\text{op}} = A_{328}$, $A_{326}^{\text{op}} = A_{326}$, $A_{335}^{\text{op}} \underset{\text{s/s}}{\sim} A_{356}$, $A_{340}^{\text{op}} = A_{348}$, $A_{345}^{\text{op}} = A_{355}$, $A_{346}^{\text{op}} = A_{364}$, $A_{349}^{\text{op}} = A_{360}$, $A_{350}^{\text{op}} = A_{357}$, $A_{351}^{\text{op}} \underset{\text{s/s}}{\sim} A_{353}$, $A_{354}^{\text{op}} = A_{371}$, $A_{362}^{\text{op}} = A_{365}$, $A_{367}^{\text{op}} = A_{375}$, $A_{368}^{\text{op}} = A_{372}$, $A_{369}^{\text{op}} = A_{378}$, $A_{373}^{\text{op}} = A_{373}$, $A_{374}^{\text{op}} = A_{381}$, $A_{379}^{\text{op}} = A_{380}$, $A_{382}^{\text{op}} = A_{383}$, $A_{386}^{\text{op}} = A_{390}$, $A_{389}^{\text{op}} = A_{389}$

A_{18}	$(2;1,7)$	$\underset{\text{der}}{\sim}$	A_{216}	$(18)(274635)$	$\Rightarrow$	$A_{18} \underset{\text{der}}{\sim} A_{240}$
$A_{51}(*)$	$(2;1,7)$	$\underset{\text{der}}{\sim}$	A_{281}	$(12)(678)$	$\Rightarrow$	$A_{69} \underset{\text{der}}{\sim} A_{267}$
$A_{51}(*)$	$(2;1,7),(4;3,8)$	$\underset{\text{der}}{\sim}$	A_{252}	$(18)(27654)$	$\Rightarrow$	$A_{69} \underset{\text{der}}{\sim} A_{266}$
A_{51}	$(6;5)$	$\underset{\text{der}}{\sim}$	A_{125}	(687)	$\Rightarrow$	$A_{69} \underset{\text{der}}{\sim} A_{159}$
A_{51}	$(6;5),(8;5)$	$\underset{\text{der}}{\sim}$	A_{71}	(568)	$\Rightarrow$	$A_{69} \underset{\text{der}}{\sim} A_{65}$
$A_{74}(*)$	$(3;2,8)$	$\underset{\text{der}}{\sim}$	A_{318}	$(1236)(78)$	$\Rightarrow$	$A_{78} \underset{\text{der}}{\sim} A_{315}$
A_{74}	$(4;3)$	$\underset{\text{der}}{\sim}$	A_{201}	$(1654)(2783)$	$\Rightarrow$	$A_{78} \underset{\text{der}}{\sim} A_{201}$
$A_{78}(*)$	$(2;1,7,8)$	$\underset{\text{der}}{\sim}$	A_{140}	(1834567)	$\Rightarrow$	$A_{74} \underset{\text{der}}{\sim} A_{174}$
A_{78}	$(4;3)$	$\underset{\text{der}}{\sim}$	A_{166}	(16548)	$\Rightarrow$	$A_{74} \underset{\text{der}}{\sim} A_{165}$
A_{78}	$(4;3),(7;3)$	$\underset{\text{der}}{\sim}$	A_{51}	(17348)	$\Rightarrow$	$A_{74} \underset{\text{der}}{\sim} A_{69}$
$A_{78}(*)$	$(2;1,7,8),(4;3)$	$\underset{\text{der}}{\sim}$	A_{246}	(1347)	$\Rightarrow$	$A_{74} \underset{\text{der}}{\sim} A_{265}$

A_{77}	$(6;4)$	$\widetilde{\text{der}}$	A_{181}	(67)	$\Rightarrow$	$A_{86} \widetilde{\text{der}} A_{202}$
A_{77}	$(5;4)$	$\widetilde{\text{der}}$	A_{140}	(38765)	$\Rightarrow$	$A_{86} \widetilde{\text{der}} A_{174}$
A_{83}	$(2;1,6)$	$\widetilde{\text{der}}$	A_{322}	$(1827)(3654)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{328}$
A_{83}	$(6;5)$	$\widetilde{\text{der}}$	A_{212}	$(386)(475)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{220}$
A_{83}	$(7;5)$	$\widetilde{\text{der}}$	A_{216}	(78)	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{214}$
A_{83}	$(6;5),(7;5)$	$\widetilde{\text{der}}$	A_{321}	(378546)	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{327}$
A_{83}	$(4;3,8)$	$\widetilde{\text{der}}$	A_{293}	(48765)	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{263}$
A_{83}	$(4;3,8),(6;5)$	$\widetilde{\text{der}}$	A_{351}	$(386)(475)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{353}$
A_{83}	$(4;3,8),(7;5)$	$\widetilde{\text{der}}$	A_{200}	$(465)(78)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{183}$
A_{83}	$(2;1,6),(4;3,8)$	$\widetilde{\text{der}}$	A_{389}	$(18267)(35)$		
A_{83}	$(2;1,6),(7;5)$	$\widetilde{\text{der}}$	A_{382}	$(23)(456)(78)$	$\Rightarrow$	$A_{83} \widetilde{\text{der}} A_{383}$
$A_{125}(*)$	$(2;1,6)$	$\widetilde{\text{der}}$	A_{350}	$(17)(264)(35)$	$\Rightarrow$	$A_{159} \widetilde{\text{der}} A_{357}$
$A_{125}(*)$	$(2;1,6),(4;3,7)$	$\widetilde{\text{der}}$	A_{335}	$(12)(47856)$	$\Rightarrow$	$A_{159} \widetilde{\text{der}} A_{356}$
A_{140}	$(8;2,7)$	$\widetilde{\text{der}}$	A_{183}	(5768)	$\Rightarrow$	$A_{174} \widetilde{\text{der}} A_{200}$
$A_{165}(*)$	$(2;1,4)$	$\widetilde{\text{der}}$	A_{371}	$(174385)(26)$	$\Rightarrow$	$A_{166} \widetilde{\text{der}} A_{354}$
A_{165}	$(7;2)$	$\widetilde{\text{der}}$	A_{295}	$(17)(38)(456)$	$\Rightarrow$	$A_{166} \widetilde{\text{der}} A_{262}$
A_{178}	$(2;1,6)$	$\widetilde{\text{der}}$	A_{199}	(374856)	$\Rightarrow$	$A_{204} \widetilde{\text{der}} A_{203}$
A_{178}	$(5;4,8)$	$\widetilde{\text{der}}$	A_{365}	$(182637)(45)$	$\Rightarrow$	$A_{204} \widetilde{\text{der}} A_{362}$
A_{178}	$(7;6)$	$\widetilde{\text{der}}$	A_{281}	$(16358)(247)$	$\Rightarrow$	$A_{204} \widetilde{\text{der}} A_{367}$
A_{213}	$(3;2,7)$	$\widetilde{\text{der}}$	A_{379}	(346758)	$\Rightarrow$	$A_{219} \widetilde{\text{der}} A_{380}$
A_{213}	$(8;3)$	$\widetilde{\text{der}}$	A_{77}	(1)	$\Rightarrow$	$A_{219} \widetilde{\text{der}} A_{86}$
A_{223}	$(3;2,6)$	$\widetilde{\text{der}}$	A_{381}	$(34)(5876)$	$\Rightarrow$	$A_{223} \widetilde{\text{der}} A_{374}$
A_{223}	$(1;4)$	$\widetilde{\text{der}}$	A_{322}	$(1827)(35)(46)$	$\Rightarrow$	$A_{223} \widetilde{\text{der}} A_{328}$
$A_{234}(*)$	$(5;4,6,8)$	$\widetilde{\text{der}}$	A_{293}	(4685)	$\Rightarrow$	$A_{234} \widetilde{\text{der}} A_{263}$
A_{241}	$(8;5)$	$\widetilde{\text{der}}$	A_{178}	(132)	$\Rightarrow$	$A_{274} \widetilde{\text{der}} A_{204}$
A_{249}	$(4;3,6)$	$\widetilde{\text{der}}$	A_{285}	(13247568)	$\Rightarrow$	$A_{260} \widetilde{\text{der}} A_{261}$
$A_{260}(*)$	$(2;1,4,7)$	$\widetilde{\text{der}}$	A_{345}	$(134)(687)$	$\Rightarrow$	$A_{249} \widetilde{\text{der}} A_{355}$
A_{260}	$(4;3)$	$\widetilde{\text{der}}$	A_{125}	$(34)(67)$	$\Rightarrow$	$A_{249} \widetilde{\text{der}} A_{159}$
A_{279}	$(3;2,5)$	$\widetilde{\text{der}}$	A_{241}	(18274536)	$\Rightarrow$	$A_{303} \widetilde{\text{der}} A_{274}$
A_{283}	$(3;2,5,7)$	$\widetilde{\text{der}}$	A_{261}	$(18)(267)(34)$	$\Rightarrow$	$A_{296} \widetilde{\text{der}} A_{285}$
A_{297}	$(1;4)$	$\widetilde{\text{der}}$	A_{362}	$(18)(2736)$	$\Rightarrow$	$A_{310} \widetilde{\text{der}} A_{365}$
A_{306}	$(7;4,8)$	$\widetilde{\text{der}}$	A_{65}	(15437268)	$\Rightarrow$	$A_{307} \widetilde{\text{der}} A_{71}$
A_{312}	$(4;3)$	$\widetilde{\text{der}}$	A_{183}	(34)	$\Rightarrow$	$A_{314} \widetilde{\text{der}} A_{200}$
A_{326}	$(8;3,7)$	$\widetilde{\text{der}}$	A_{328}	$(172846)(35)$	$\Rightarrow$	$A_{326} \widetilde{\text{der}} A_{322}$
$A_{340}(*)$	$(6;5,7)$	$\widetilde{\text{der}}$	A_{360}	$(148)(25)$	$\Rightarrow$	$A_{348} \widetilde{\text{der}} A_{349}$
A_{360}	$(1;3)$	$\widetilde{\text{der}}$	A_{306}	$(12)(48)(576)$	$\Rightarrow$	$A_{349} \widetilde{\text{der}} A_{307}$
A_{346}	$(7;2,6)$	$\widetilde{\text{der}}$	A_{362}	(57)	$\Rightarrow$	$A_{364} \widetilde{\text{der}} A_{365}$
A_{367}	$(4;3,5)$	$\widetilde{\text{der}}$	A_{178}	$(18)(25)(3746)$	$\Rightarrow$	$A_{375} \widetilde{\text{der}} A_{204}$
A_{368}	$(8;3)$	$\widetilde{\text{der}}$	A_{283}	$(1827)(46)$	$\Rightarrow$	$A_{372} \widetilde{\text{der}} A_{296}$
A_{369}	$(5;4)$	$\widetilde{\text{der}}$	A_{386}	$(132)(46758)$	$\Rightarrow$	$A_{378} \widetilde{\text{der}} A_{390}$
A_{369}	$(7;6,8)$	$\widetilde{\text{der}}$	A_{373}	(17428536)	$\Rightarrow$	$A_{378} \widetilde{\text{der}} A_{373}$
A_{369}	$(4;3,7)$	$\widetilde{\text{der}}$	A_{354}	$(16)(287)(35)$	$\Rightarrow$	$A_{378} \widetilde{\text{der}} A_{371}$

$(*)$ the direction of some arrow(s) is changed in a sink or source

D.10 Polynomial $6(x^8+x^6+x^5+x^3+x^2+1)$

$A_{15}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{15}$, $A_{88}^{\mathrm{op}} = A_{88}$, $A_{179}^{\mathrm{op}} = A_{184}$, $A_{205}^{\mathrm{op}} = A_{211}$, $A_{209}^{\mathrm{op}} = A_{215}$, $A_{268}^{\mathrm{op}} = A_{299}$, $A_{270}^{\mathrm{op}} = A_{280}$, $A_{290}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{319}$, $A_{300}^{\mathrm{op}} = A_{308}$, $A_{309}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{317}$, $A_{313}^{\mathrm{op}} = A_{323}$, $A_{320}^{\mathrm{op}} = A_{320}$, $A_{325}^{\mathrm{op}} \underset{\text{s/s}}{\sim} A_{325}$, $A_{331}^{\mathrm{op}} = A_{339}$, $A_{341}^{\mathrm{op}} = A_{358}$, $A_{376}^{\mathrm{op}} = A_{377}$, $A_{384}^{\mathrm{op}} = A_{385}$, $A_{387}^{\mathrm{op}} = A_{391}$

A_{15}^{op}	$(2;1,3)$	$\widetilde{\text{der}}$	A_{184}	$(28)(37)(46)$	$\Rightarrow$	$A_{15} \widetilde{\text{der}} A_{179}$
A_{15}^{op}	$(2;1,3),(7;8)$	$\widetilde{\text{der}}$	A_{280}	$(12)(378)(46)$	$\Rightarrow$	$A_{15} \widetilde{\text{der}} A_{270}$
A_{184}	$(7;6,8)$	$\widetilde{\text{der}}$	A_{215}	$(123658)(47)$	$\Rightarrow$	$A_{179} \widetilde{\text{der}} A_{209}$

A_{280}	$(7;1,6)$	$\underset{\text{der}}{\sim}$	A_{308}	(4657)	$\Rightarrow$	$A_{270} \underset{\text{der}}{\sim} A_{300}$
A_{308}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{313}	$(1234)(67)$	$\Rightarrow$	$A_{300} \underset{\text{der}}{\sim} A_{323}$
A_{88}	$(2;1,8)$	$\underset{\text{der}}{\sim}$	A_{323}	$(124)(5678)$	$\Rightarrow$	$A_{88} \underset{\text{der}}{\sim} A_{313}$
A_{88}	$(2;1,8),(5;3)$	$\underset{\text{der}}{\sim}$	A_{377}	$(17485)(326)$	$\Rightarrow$	$A_{88} \underset{\text{der}}{\sim} A_{376}$
A_{384}	$(4;3,6,8)$	$\underset{\text{der}}{\sim}$	A_{300}	(34)	$\Rightarrow$	$A_{385} \underset{\text{der}}{\sim} A_{308}$
A_{325}	$(2;1,4,8)$	$\underset{\text{der}}{\sim}$	A_{323}	$(1468)(23)(57)$	$\Rightarrow$	$A_{325} \underset{\text{der}}{\sim} A_{313}$
A_{317}^{op}	$(5;2,6)$	$\underset{\text{der}}{\sim}$	A_{325}	(386547)	$\Rightarrow$	$A_{309} \underset{\text{der}}{\sim} A_{325}$
A_{211}	$(3;2,8)$	$\underset{\text{der}}{\sim}$	A_{377}	$(142536)(78)$	$\Rightarrow$	$A_{205} \underset{\text{der}}{\sim} A_{376}$
A_{205}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{341}	(16542738)	$\Rightarrow$	$A_{211} \underset{\text{der}}{\sim} A_{358}$
A_{339}	$(6;1,5)$	$\underset{\text{der}}{\sim}$	A_{358}	(476)	$\Rightarrow$	$A_{331} \underset{\text{der}}{\sim} A_{341}$
A_{387}	$(4;3,7,8)$	$\underset{\text{der}}{\sim}$	A_{377}	(1425786)	$\Rightarrow$	$A_{391} \underset{\text{der}}{\sim} A_{376}$
A_{299}	$(5;4,8)$	$\underset{\text{der}}{\sim}$	A_{391}	$(186)(243)$	$\Rightarrow$	$A_{268} \underset{\text{der}}{\sim} A_{387}$
A_{290}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{309}	$(18)(267)(34)$	$\Rightarrow$	$A_{319} \underset{\text{der}}{\sim} A_{317}$
A_{320}	$(7;6,8)$	$\underset{\text{der}}{\sim}$	A_{319}	(14725836)	$\Rightarrow$	$A_{320} \underset{\text{der}}{\sim} A_{290}$

D.11 Polynomial $6(x^8 + x^7 + 2x^4 + x + 1)$

$A_{38}^{\text{op}} = A_{45}$, $A_{39}^{\text{op}} \underset{\text{s/s}}{\sim} A_{73}$, $A_{50}^{\text{op}} \underset{\text{s/s}}{\sim} A_{57}$, $A_{56}^{\text{op}} \underset{\text{s/s}}{\sim} A_{68}$, $A_{62}^{\text{op}} = A_{75}$, $A_{97}^{\text{op}} = A_{115}$, $A_{108}^{\text{op}} = A_{114}$, $A_{117}^{\text{op}} = A_{188}$, $A_{138}^{\text{op}} = A_{189}$, $A_{139}^{\text{op}} = A_{195}$, $A_{141}^{\text{op}} \underset{\text{s/s}}{\sim} A_{164}$, $A_{145}^{\text{op}} = A_{158}$, $A_{147}^{\text{op}} = A_{198}$, $A_{156}^{\text{op}} = A_{167}$, $A_{157}^{\text{op}} = A_{190}$, $A_{180}^{\text{op}} = A_{197}$, $A_{191}^{\text{op}} = A_{194}$, $A_{210}^{\text{op}} = A_{217}$, $A_{228}^{\text{op}} = A_{248}$, $A_{229}^{\text{op}} = A_{264}$, $A_{230}^{\text{op}} = A_{233}$, $A_{235}^{\text{op}} = A_{251}$, $A_{245}^{\text{op}} \underset{\text{s/s}}{\sim} A_{254}$, $A_{253}^{\text{op}} = A_{255}$, $A_{256}^{\text{op}} = A_{287}$, $A_{257}^{\text{op}} \underset{\text{s/s}}{\sim} A_{292}$, $A_{284}^{\text{op}} \underset{\text{s/s}}{\sim} A_{294}$, $A_{289}^{\text{op}} = A_{298}$, $A_{291}^{\text{op}} = A_{316}$, $A_{336}^{\text{op}} = A_{347}$, $A_{344}^{\text{op}} = A_{359}$

A_{38}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{233}	$(1728)(3645)$	$\Rightarrow$	$A_{45} \underset{\text{der}}{\sim} A_{230}$
A_{45}	$(6;5,8)$	$\underset{\text{der}}{\sim}$	A_{57}	$(18)(2536)(47)$	$\Rightarrow$	$A_{38} \underset{\text{der}}{\sim} A_{50}$
A_{68}	$(6;3,8)$	$\underset{\text{der}}{\sim}$	A_{39}	$(18)(27456)$	$\Rightarrow$	$A_{56} \underset{\text{der}}{\sim} A_{73}$
A_{75}	$(3;2,8)$	$\underset{\text{der}}{\sim}$	A_{235}	(1537246)	$\Rightarrow$	$A_{62} \underset{\text{der}}{\sim} A_{251}$
A_{62}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{289}	(1524876)	$\Rightarrow$	$A_{75} \underset{\text{der}}{\sim} A_{298}$
A_{108}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{145}	$(1845)(2736)$	$\Rightarrow$	$A_{114} \underset{\text{der}}{\sim} A_{158}$
A_{114}	$(8;6)$	$\underset{\text{der}}{\sim}$	A_{228}	$(185)(26)(37)$	$\Rightarrow$	$A_{108} \underset{\text{der}}{\sim} A_{248}$
A_{114}	$(5;4,7),(8;6)$	$\underset{\text{der}}{\sim}$	A_{38}	$(123)(578)$	$\Rightarrow$	$A_{108} \underset{\text{der}}{\sim} A_{45}$
A_{115}	$(4;3,7),(6;5)$	$\underset{\text{der}}{\sim}$	A_{45}	$(1728)(36)(45)$	$\Rightarrow$	$A_{97} \underset{\text{der}}{\sim} A_{38}$
A_{117}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{158}	(134)	$\Rightarrow$	$A_{188} \underset{\text{der}}{\sim} A_{145}$
$A_{39}(*)$	$(7;6,8)$	$\underset{\text{der}}{\sim}$	A_{257}	(1)	$\Rightarrow$	$A_{73} \underset{\text{der}}{\sim} A_{259}$
$A_{68}(*)$	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{287}	(16347258)	$\Rightarrow$	$A_{56} \underset{\text{der}}{\sim} A_{256}$
A_{189}	$(3;2,4)$	$\underset{\text{der}}{\sim}$	A_{197}	$(1432)(687)$	$\Rightarrow$	$A_{138} \underset{\text{der}}{\sim} A_{180}$
A_{139}	$(5;3)$	$\underset{\text{der}}{\sim}$	A_{257}	$(58)(67)$	$\Rightarrow$	$A_{195} \underset{\text{der}}{\sim} A_{292}$
A_{139}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{180}	(134)	$\Rightarrow$	$A_{195} \underset{\text{der}}{\sim} A_{197}$
A_{139}	$(5;3),(7;6,8)$	$\underset{\text{der}}{\sim}$	A_{57}	(57)	$\Rightarrow$	$A_{195} \underset{\text{der}}{\sim} A_{50}$
A_{141}	$(2;1,3,6)$	$\underset{\text{der}}{\sim}$	A_{253}	$(24)(35)$	$\Rightarrow$	$A_{164} \underset{\text{der}}{\sim} A_{255}$
A_{141}	$(5;4)$	$\underset{\text{der}}{\sim}$	A_{235}	$(17)(246358)$	$\Rightarrow$	$A_{164} \underset{\text{der}}{\sim} A_{251}$
A_{147}	$(1;4)$	$\underset{\text{der}}{\sim}$	A_{287}	(1638257)	$\Rightarrow$	$A_{198} \underset{\text{der}}{\sim} A_{256}$
$A_{156}(*)$	$(6;4,8)$	$\underset{\text{der}}{\sim}$	A_{359}	$(123)(67)$	$\Rightarrow$	$A_{167} \underset{\text{der}}{\sim} A_{344}$
A_{344}	$(3;2,5)$	$\underset{\text{der}}{\sim}$	A_{210}	(146532)	$\Rightarrow$	$A_{359} \underset{\text{der}}{\sim} A_{217}$
A_{359}	$(4;3,7)$	$\underset{\text{der}}{\sim}$	A_{189}	(5687)	$\Rightarrow$	$A_{344} \underset{\text{der}}{\sim} A_{138}$
A_{157}	$(4;2,7)$	$\underset{\text{der}}{\sim}$	A_{347}	$(123)(45)(687)$	$\Rightarrow$	$A_{190} \underset{\text{der}}{\sim} A_{336}$
A_{336}	$(5;4,7)$	$\underset{\text{der}}{\sim}$	A_{141}	$(17436)(285)$	$\Rightarrow$	$A_{347} \underset{\text{der}}{\sim} A_{164}$
A_{191}	$(5;4,7,8)$	$\underset{\text{der}}{\sim}$	A_{230}	$(1837)(25)(46)$	$\Rightarrow$	$A_{194} \underset{\text{der}}{\sim} A_{233}$
A_{229}	$(2;1,4)$	$\underset{\text{der}}{\sim}$	A_{245}	(134)	$\Rightarrow$	$A_{264} \underset{\text{der}}{\sim} A_{254}$
A_{229}	$(7;6)$	$\underset{\text{der}}{\sim}$	A_{139}	(67)	$\Rightarrow$	$A_{264} \underset{\text{der}}{\sim} A_{195}$
A_{284}	$(4;3,5)$	$\underset{\text{der}}{\sim}$	A_{75}	(387654)	$\Rightarrow$	$A_{294} \underset{\text{der}}{\sim} A_{62}$
A_{291}	$(2;1,5)$	$\underset{\text{der}}{\sim}$	A_{62}	$(35)(46)$	$\Rightarrow$	$A_{316} \underset{\text{der}}{\sim} A_{75}$
A_{291}	$(1;3)$	$\underset{\text{der}}{\sim}$	A_{210}	$(164)(253)$	$\Rightarrow$	$A_{316} \underset{\text{der}}{\sim} A_{217}$
$A_{291}(*)$	$(3;2,4,6)$	$\underset{\text{der}}{\sim}$	A_{284}	(145632)	$\Rightarrow$	$A_{316} \underset{\text{der}}{\sim} A_{294}$

(∗) the direction of some arrow(s) is changed in a sink or source

D.12 Polynomial $8(x^8+2x^7+2x^4+2x+1)$

$A_{91}^{\mathrm{op}} \underset{\mathrm{s/s}}{\sim} A_{101}$

A_{91}	$(6;5,8)$	$\underset{\mathrm{der}}{\sim}$	A_{101}	(5786)

D.13 Polynomial $8(x^8+x^7+x^6+2x^4+x^2+x+1)$

$A_{59}^{\mathrm{op}} = A_{63}$, $A_{64}^{\mathrm{op}} = A_{82}$, $A_{79}^{\mathrm{op}} = A_{81}$, $A_{130}^{\mathrm{op}} = A_{168}$, $A_{236}^{\mathrm{op}} = A_{288}$, $A_{244}^{\mathrm{op}} = A_{271}$, $A_{250}^{\mathrm{op}} = A_{259}$, $A_{269}^{\mathrm{op}} = A_{301}$, $A_{329}^{\mathrm{op}} = A_{334}$, $A_{330}^{\mathrm{op}} = A_{332}$

A_{59}	$(6;3,8)$	$\underset{\mathrm{der}}{\sim}$	A_{64}	(1745628)	$\Rightarrow$	$A_{63} \underset{\mathrm{der}}{\sim} A_{82}$
A_{63}	$(4;2,8)$	$\underset{\mathrm{der}}{\sim}$	A_{288}	$(123)(45)(678)$	$\Rightarrow$	$A_{59} \underset{\mathrm{der}}{\sim} A_{236}$
A_{79}	$(2;1,6)$	$\underset{\mathrm{der}}{\sim}$	A_{250}	$(1827)(35)(46)$	$\Rightarrow$	$A_{81} \underset{\mathrm{der}}{\sim} A_{259}$
A_{244}	$(3;2,5)$	$\underset{\mathrm{der}}{\sim}$	A_{259}	$(18)(273645)$	$\Rightarrow$	$A_{271} \underset{\mathrm{der}}{\sim} A_{250}$
A_{269}	$(6;1,5)$	$\underset{\mathrm{der}}{\sim}$	A_{236}	(1735428)	$\Rightarrow$	$A_{301} \underset{\mathrm{der}}{\sim} A_{288}$
A_{330}	$(6;3,7)$	$\underset{\mathrm{der}}{\sim}$	A_{130}	(1)	$\Rightarrow$	$A_{332} \underset{\mathrm{der}}{\sim} A_{168}$
A_{334}	$(1;3,5)$	$\underset{\mathrm{der}}{\sim}$	A_{168}	(1423)	$\Rightarrow$	$A_{329} \underset{\mathrm{der}}{\sim} A_{130}$
A_{79}	$(2;1,6),(4;3)$	$\underset{\mathrm{der}}{\sim}$	A_{330}	$(1827)(35)$	$\Rightarrow$	$A_{81} \underset{\mathrm{der}}{\sim} A_{332}$
A_{63}	$(4;2,8),(6;5)$	$\underset{\mathrm{der}}{\sim}$	A_{334}	$(18)(2645)(37)$	$\Rightarrow$	$A_{59} \underset{\mathrm{der}}{\sim} A_{329}$
A_{64}	$(2;1,4)$	$\underset{\mathrm{der}}{\sim}$	A_{82}	(134)		

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