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Spherical homogeneous spaces and their equivariant embeddings

Lectures for the Winter School
Homogeneous Spaces and Geometric Representation Theory
February 27 – March 2, 2012
Ruhr University Bochum

Spherical homogeneous spaces are both classical and modern objects of study in algebra and geometry. Particular examples were studied by geometers since the XIX-th century, starting from spheres and projective spaces and passing to Grassmannians, flag varieties, spaces of quadrics and, more generally, symmetric spaces. However, the unity of properties and approaches to the study of spherical spaces was well understood not too long ago, which led to an active development of the theory during last 30 years.

Spherical homogeneous spaces lie at the crossroads of algebraic geometry, theory of algebraic groups, enumerative geometry, harmonic analysis, and representation theory. We shall consider various properties of spherical spaces from viewpoints of algebraic transformation groups, harmonic analysis, and equivariant symplectic geometry. By standard reasons of algebraic geometry, in order to solve various problems on a spherical homogeneous space it is helpful to compactify it keeping track of the group action, i.e., to consider equivariant completions or, more generally, open embeddings of a given homogeneous space, called spherical varieties. Toric varieties are a special case. Alike the toric case, the classification and study of spherical varieties relies on certain data of combinatorial nature from discrete and convex geometry: lattices, valuation cones and colors, colored cones and fans, piecewise linear functions and polytopes, etc. We shall develop this theory and pass to applications, which include: theory of divisors and line bundles on spherical varieties, solving enumerative problems on spherical homogeneous spaces, problems in representation theory such as tensor product decompositions, algebraic semigroups, etc. If time allows, we shall speak on the recent progress in the classification of spherical homogeneous spaces based on the concepts of a wonderful variety and a spherical system.