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Parity sheaves

Lectures for the Winter School
Homogeneous Spaces and Geometric Representation Theory
February 27 – March 2, 2012
Ruhr University Bochum

In geometric representation theory a central role is played by the decomposition theorem, which asserts the semi-simplicity of the direct image of an intersection cohomology complex under any proper map. The decomposition theorem is only valid when the coefficients of the sheaves are of characteristic zero, and indeed it is easy to give examples with positive characteristic coefficients where the analogous statement is no longer valid.

However, for applications in modular representation theory it is desirable to understand this "failure" of the decomposition theorem, and to have some replacement when this "failure" occurs. This project is probably too ambitious in general. However the varieties which one meets in representation theory (Schubert varieties, the nilpotent cone etc.) often have special features which one can hope to exploit to develop a theory.

This course will give an introduction to the theory of parity sheaves, which provide one way of understanding the failure of the decomposition theorem in geometric representation theory. Where possible I will explain links to representation theory, so that one gets a sense of the rich interplay between the topology of complex algebraic maps and (modular) representation theory.

This course will consist of four lectures. The following is a rough guide only:

Lecture 1: An overview of the derived category of constructible sheaves: intersection cohomology sheaves, examples of the decomposition theorem (smooth maps, maps between complex surfaces).

Lecture 2: An introduction to parity sheaves. Their definition and uniqueness. The example of the flag variety. A brief introduction to the equivariant derived category. Parity sheaves on toric varieties, nilpotent cones and quiver varieties.

Lecture 3: Examples of the failure of the decomposition theorem. Deciding the multiplicities of parity sheaves in a direct image terms of intersection forms. Examples from toric geometry and flag varieties.

Lecture 4: Overview of the Springer correspondence in characteristic 0. A reminder about the modular representation theory of the symmetric group. The modular Springer correspondence for the symmetric group and parity sheaves.