Mathematisches Kolloquium

Symmetries in Algebra and Geometry

Prof. Hanspeter Kraft (Basel)

The concept of symmetry plays an important role in many areas of mathematics. We might ask how much information about a mathematical object can be retrieved from its symmetries. Here, a mathematical object can be many things: a geometric figure, a topological space, a manifold, an algebraic structure, an algebraic curve, a Riemannian surface, an algebraic variety, etc. For example, a connected surface in the space which allows the full rotation group as symmetries is a sphere. We also know that the Platonic solids are described by their symmetries. A little known theorem in this context comes from Richard P. Filipkewicz (1982): A differentiable manifold is determined up to diffeomorphism by its automorphism group. Finally, one can understand Felix Klein's famous "Erlanger Programm" that one should study and describe geometrical objects about their symmetry properties.

For algebraic objects such as complex varieties, not much is known today. One of the reasons might be that the automorphism group of a variety is and remains a mysterious object. This already begins with the automorphism groups of complex affine n–space. In this context, Shararevich proposed in 1966 to give this group the structure of an "infinite–dimensional" algebraic group, and he proved some interesting properties, e.g. the existence of a Lie algebra. His papers did not get much attention at that time, and this theme was only taken up again recently, in connection with new developments in affine algebraic geometry.

In my lecture, I will focus on the algebraic side of the problem and will give an overview of what we know today, what we would like to know and where the difficulties lie.