Mathematisches Kolloquium

On a combinatorial theory of valuations on polytopes

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The confluence of Minkowski addition and volume gives rise to the vast theory of mixed volumes and intrinsic volumes. Mixed volumes and the related geometric inequalities have applications in many areas including algebra and combinatorics. A celebrated result of Hadwiger asserts that the intrinsic volumes constitute a basis for the continuous and rigid-motion invariant valuations on convex bodies and, moreover, this basis is distinguished by nonnegativity- and monotonicity properties. This is a quite complete and mathematically appealing situation for general convex bodies.

In the discrete setting (i.e. lattice polytopes), discrete volume (i.e. counting lattice points) takes the place of the volume. But whereas the structure of the space of lattice-invariant valuations on lattice polytopes parallels the continuous situation above quite remarkably, our understanding of nonnegative and monotone valuations is simply unsatisfactory. Even worse, the nonnegativity and monotonicity of the mixed volumes is genuinely lost for the discrete mixed volumes.

In this talk I will advocate a combinatorial theory of nonnegative, monotone, and mixed valuations. In the continuous case, this underlines the prominent role played by volume and recovers the classical mixed volumes. For lattice polytopes, we obtain a Hadwiger-type theorem and we show that our notion of combinatorial mixed valuation preserves nonnegativity and monotonicity for many valuations including the discrete volume. I will emphasize the strong ties to the combinatorics of subdivisions of (lattice) polytopes and, if time permits, I will remark on applications to recent results of DiRocco, Haase, and Nill on the (motivic) arithmetic genus. The talk is based on joint work with Katharina Jochemko.