LARGE VALUES OF EIGENFUNCTIONS AND ARITHMETIC HYPERBOLIC 3-MANIFOLDS OF MACLACHLAN-REID TYPE

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Abstract

One aspect of quantum chaos are extreme values of high-energy eigenfunctions, which, on Riemannian manifolds of negative curvature, are not well understood and can depend heavily on the global geometry of the manifold.

The principal result to be presented shows that there is a distinguished class of arithmetic hyperbolic 3-manifolds on which a sequence of L^2-normalized high-energy Hecke-Maass eigenforms achieve values as large as a power of the Laplacian eigenvalue. Power growth, first observed in this context by Rudnick and Sarnak, is (by far) not expected generically and stands in stark contrast with the statistical models suggested by the so-called random wave conjecture.

The arithmetic hyperbolic 3-manifolds on which the exceptional behavior is exhibited can be characterized in terms of their invariant quaternion algebras and are, up to commensurability, precisely those containing immersed totally geodesic surfaces, as described by Maclachlan and Reid. The question of identifying the Hecke-Maass eigenforms achieving power growth will also be discussed through representation theory.