Arithmetic of marked order polytopes
(Katharina Jochemko, 10.02.2015, Köln)

A map f from a poset P into a poset Q is order preserving if $f(p) \Leftarrow f(q)$ in Q whenever $p < q$ in P, and the inequality is strict for strictly order preserving maps. Stanley considered the problem of counting order preserving maps from a finite poset P into a chain of length n. He showed that for a fixed poset P the number of order preserving maps is given by a polynomial in n and gave an interpretation for evaluating this polynomial at negative integers in terms of strictly order preserving maps.
We consider a more general problem: Given a finite poset P, a subposet A and a map f from A into the integers. What is the number of integer-valued order preserving maps with domain P extending f? We take the geometric route by passing to real-valued maps and study marked order polytopes. We show that the function counting integer-valued extensions is a piecewise polynomial in the values of f and give an interpretation for the evaluation at order reversing maps. We apply these results to Gelfand-Tsetlin patterns and give a geometric proof of a combinatorial reciprocity for monotone triangles due to Fischer and Riegler (2011).
This is joint work with Raman Sanyal.