Title: Degenerating Grassmannians into toric varieties
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Abstract: A toric variety is a certain algebraic variety modeled on a convex polyhedron. Toric varieties play an important role in commutative algebra. I will talk about the method of degenerating the Grassmannian $\mathrm{Gr}(d, n)$ into toric varieties. The Grassmannian $\mathrm{Gr}(d, n)$ is the subvariety of the projective space $\mathbb{P}^{\binom{n}{d}-1}$ consisting of all d-dimensional subspaces of $\mathbb{K}^n$ (defined by the Plücker ideal). There are many toric degenerations and integrable systems for Grassmannians $\mathrm{Gr}(2, n)$ are encoded by combinatorics of trees, equivalently subdivisions of polygons. These degenerations can also be seen to arise from top dimensional cones of the tropicalisation of the Grassmannian. A point in the tropical Grassmannian arises from a classical plane defined by a rank d matrix of size $d \times n$. The top dimensional cones of the tropical Grassmannian are good candidates for toric degenerations via initial ideals of the Plücker ideal. For the Grassmannians $\mathrm{Gr}(2, n)$ each top dimensional cone of the tropical Grassmannian gives a toric degeneration. But for $\mathrm{Gr}(3, n)$ the situation is not as nice, and there are cones of the tropical Grassmannian which do not produce toric degenerations. However, each top dimensional cone of the tropical $\mathrm{Gr}(3, n)$ determines a combinatorial tree arrangement. In an ongoing joint work with Kristin Shaw, we identify which cones of the tropical $\mathrm{Gr}(3, n)$ produce toric degenerations in terms of the combinatorics of their associated tree arrangements.