The $(sp(4), sl(2))$-modules of finite type

A (g, k)-module of a generic pair of Lie algebras is a straight-forward generalization of a (g, k)-module for a symmetric pair of Lie algebras, i.e. of Harish-Chandra modules. It turns out that to achieve a reasonable theory of (g, k)-modules one has to introduce additional condition(s) on them. One such a condition, being of finite type, was introduced by I. Penkov and G. Zuckerman and is a straightforward generalization of a well-known admissibility condition for Harish-Chandra modules.

If k is a symmetric subalgebra of g, k has finitely many orbits on the full flag variety of g and this is one of the key points in the description of (g, k)-modules for symmetric pairs (g, k), i.e. of Harish-Chandra modules. The smallest pair of semi-simple Lie algebras for which this finitness condition fails (say, by dimension reasons) is $k = sl(2)$ in $sp(4) = g$.

In my talk I will provide some description of $(sp(4), sl(2))$-modules (of finite type) via the description of the (infinite) set of $SL(2)$-orbits on the full flag variety of $sp(4)$.