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Characterisation of the affine space by its automorphism group

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This is joint work with Hanspeter Kraft (University of Basel) and Andriy Regeta (University of Cologne). The main problem we address in this talk is the characterization of the affine space $\mathbb{A}^n$ by its automorphism group $\mathrm{Aut}(\mathbb{A}^n)$. More precisely, we ask, whether the existence of an abstract group isomorphism $\mathrm{Aut}(X) \simeq \mathrm{Aut}(\mathbb{A}^n)$ implies the existence of an isomorphism of algebraic varieties $X \simeq \mathbb{A}^n$. This cannot be true in general, as for example $\mathrm{Aut}(\mathbb{A}^n) \simeq \mathrm{Aut}(\mathbb{A}^n \times V)$ for any complete variety V with trivial automorphism group. We give an affirmative answer under certain conditions on the Euler-characteristic $\chi(X)$ and the Picard group $\mathrm{Pic}(X)$:

Main Theorem. *Let X be a quasi-projective irreducible variety such that $\mathrm{Aut}(X) \simeq \mathrm{Aut}(\mathbb{A}^n)$. Then $X \simeq \mathbb{A}^n$ if one of the following conditions holds.*

(1) *X is smooth, $\chi(X) \neq 0$, $\mathrm{Pic}(X)$ is finite, and $\dim X \leq n$;*

(2) *X is toric, quasi-affine, and $\dim X \geq n$.*

After giving a brief history on some related results that concern the characterisation of geometric objects via their automorphisms, we give the key ideas of the proof of our main result.