Working Group Seminar SS 2014 - Modular Representation Theory

Arbeitsgruppe Algebra & Darstellungstheorie

April 3, 2014

In this seminar, we want to understand the basics of the representation theory of finite groups in characteristic dividing the group order (we will not deal with the interplay with characteristic zero or with the theory of characters!). Our main goal is to understand the formulation of Brauer's main theorems and their proofs. In the latter part of the seminar, we will also discuss the structure of the group algebra with a cyclic defect group due to Dade. At the end of this term, we want to be able to understand the conjectures in modular representation theory (at least those not involving characters, e.g. Broué's abelian defect group conjecture).
A knowledge of the representation theory of finite groups in characteristic zero is not needed, but might be helpful for motivational purposes. What is needed is a solid understanding of the basic representation theory of finite dimensional algebras (e.g. simple, indecomposable, projective and injective modules, Krull-Remak-Schmidt theorem, length, Jordan-Hölder theorem, ...).

1 Basic group representation theory in characteristic p

The speaker should start by recalling Maschke's theorem and its proof. The talk should include a discussion of Brauer's proof determining the number of simple modules in terms of the number of conjugacy classes (and then apply this to the simple modules of a p-group). Finally the speaker should discuss the simple modules for group algebras of cyclic groups and $SL_2(\mathbb{F}_p)$.

2 The stable module category

The speaker should prove that a module over the group algebra is projective if and only if it is injective. Afterwards (s)he should introduce the stable module category and discuss its triangulated structure. The relationship between homomorphism spaces and Ext groups should also be discussed and as an example one might like to consider the cyclic group of order two. At the end, one might want to mention the description of the stable category as a singularity category due to Buchweitz and Rickard.

3 Induction, restriction, ...

This talk should remind us of the tensor product of two representations and then proceed to define and discuss induction and restriction functors and their various properties (including the adjunction). Mackey's theorem relating indcution and restriction with different groups should be included. Finally, the speaker should also present Clifford's theorem and Green's indecomposability theorem.

4 Green correspondence (two talks)

These talks should contain a discussion of vertices and sources. Then the Green correspondence for trivial intersection groups should be formulated and it should be restated as an equivalence of stable categories (no proofs here). Then the general Green correspondence should be formulated and proved and then the same thing should be done for the Green correspondence for morphisms. Finally it should be explained how this implies the Green correspondence for trivial intersections. As an example, the speakers might want to present the Green correspondence for the simple $SL_2(\mathbb{F}_p)$-modules.

5 Blocks and defect groups

This talk should contain a detailed discussion of blocks and defect groups following chapter 13 of [1]. As an example, the blocks and defect groups for $SL_2(\mathbb{F}_p)$ should be worked out.

6 Brauer's three main theorems (two talks)

The speakers should begin with a description of the Brauer correspondence and Brauer's first main theorem. Then the relationship between the Brauer correspondence and the Green correspondence should be discussed (i.e. Brauer's second main theorem). Afterwards ... . Finally, Brauer's third main theorem should be presented and the whole discussion should be made concrete by considering the case of $SL_2(\mathbb{F}_p)$.

7 Cyclic defect groups (two talks)

In these talks, we want to understand the structure of all indecomposable projectives of blocks with cyclic defect group. The speakers should introduce Brauer tree algebras and give examples and then follow the strategy described in chapter 17 of [1]. It is unrealistic (and probably not even helpful) to include all details, so the speakers should probably concentrate on showing that blocks with cyclic defect groups are Brauer graph algebras and only state the more precise statement of being a Brauer tree algebra with certain multiplicities.

References

[1] J.L. Alperin, Local representation theory, Cambridge studies in advanced mathematics 11, CUP(2004).