

Simple Gelfand-Tsetlin modules over $\mathfrak{gl}(n, \mathbb{C})$

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The enveloping algebra of $\mathfrak{gl}(n, \mathbb{C})$ contains a large commutative subalgebra Γ called the Gelfand-Tsetlin subalgebra. A Gelfand-Tsetlin module is a $\mathfrak{gl}(n, \mathbb{C})$ -module on which the action of Γ is locally finite. At the end of the 80s Futorny and collaborators began with the classification of simple Gelfand-Tsetlin modules. They proved that for each character $\chi : \Gamma \rightarrow \mathbb{C}$ there are at most finitely many isomorphism classes of simple Gelfand-Tsetlin modules with a vector v of generalized Γ -eigenvalue χ , and that for a generic character there is only one such module. Furthermore, following Gelfand and Tsetlin's example, they give an effective presentation of this simple module, i.e. a basis and the corresponding matrix coefficients for generators of $\mathfrak{gl}(n, \mathbb{C})$. However the nongeneric case remained untreated for the next fifteen years.

In this talk I will give an introduction to the subject and discuss results appeared in the last two years regarding nongeneric Gelfand-Tsetlin modules, which point to the complete classification of simple objects in this category.