Universität zu Köln

Mathematisch-Naturwissenschaftliche Fakultät

Conference
"Universal Structures in Probabilistic Models"

Wednesday, Sept 28th — Friday, Sept 30th, 2022
Geo/Bio Lecture Hall (Room HS)
Building 310c
Zülpicher Straße 49a
50937 Köln

Program

Wednesday, September 28th

12:00 – 14:00 lunch break

14:00 – 14:45 Pierre-François Rodriguez (Imperial College London)

Title: Scaling in low-dimensional long-range percolation models

The talk will review recent progress towards understanding the critical behavior of 3-dimensional percolation models exhibiting long-range correlations. The results rigorously exhibit the scaling behavior of various observables of interest and are consistent with scaling theory below the upper-critical dimension (expectedly equal to 6).

14:50 – 15:35 Lisa Hartung (U Mainz)

Title: The speed of invasion of an advancing population

Abstract: We derive rigorous estimates on the speed of invasion of an advantageous trait in a spatially advancing population in the context of a system of one-dimensional coupled $F-KPP$ equations. The model was introduced and studied heuristically and numerically in a paper by Venegas-Ortiz et al. In that paper, it was noted that the speed of invasion by the mutant trait is faster faster when the resident population ist expanding in space compared to the speed when the resident population is already present everywhere. We use probabilistic methods, in particular the Feynman-Kac representation, to provide rigorous estimates that confirm these predictions. Based on joint work in progress with A. Bovier.

15:35 – 16:00 coffee break

16:00 – 16:45 Volker Betz (TU Darmstadt)

Title: Effective mass of the polaron - a quantitative lower bound.

Abstract: The Fröhlich polaron models a quantum particles interacting with the phonons of a polar crystal. Physically, one expects that the coupling decreases the response of the particle to external forces, leading to an increased effective mass. An important question is the behaviour of the effective mass as a function of the coupling strength α. Based on semiclassical arguments by Landau and Peklar, it is conjectured to behave like α^4 with an explicit prefactor. In this talk, I will give the first quantitative lower bound on the effective mass, showing that it grows at least like $\alpha^{2/5}$. The result is obtained by using the Feynman-Kac representation of the polaron and applying a transformation due to Mukherjee and Varadhan, which leads to a process of interacting finite intervals on the real line. Renewal theory and some explicit calculations then give the result.

16:50 – 17:35 Robert MacKay (U Warwick)

Title: Persistence of spectral projections for stochastic operators on large tensor products

Abstract: For families of stochastic operators on a countable tensor product, depending smoothly on parameters, we prove that any spectral projection persists smoothly, where smoothness is defined using norms based on ideas of Dobrushin. A rigorous perturbation theory for families of stochastic operators with spectral gap is thereby created. It is illustrated by deriving an effective slow 2-state dynamics for a 3-state probabilistic cellular automaton. Applications to metastability are expected.

17:45 – 19:00 Wine & Cheese

Thursday, September 29th

9:00 – 9:45 Peter Gracar (U Cologne)

Title: Weight-Dependent Random Connection Models: Ultrasmallness and percolation

Abstract: We investigate a large class of random graphs on the points of a Poisson process in d-dimensional space, d>=1, which combine scale-free degree distributions and long-range effects. Every Poisson point carries an independent random mark and given the mark and position of the points we form an edge between two points independently with a probability depending on the two marks and the distance of the points. We show that for a large class of graphs the graph distance between two points is of doubly logarithmic order in their euclidean distance whenever the mark distribution and spatial embedding satisfy certain conditions. We will also see that under these same conditions, the graph is robust in the sense that its critical bond percolation probability equals 0. Joint work with Arne Grauer, Lukas Lüchtrath and Peter Mörters.

9:50 – 10:35 Eveliina Peltola (U Bonn & Aalto University)

Title: On crossing probabilities in critical random-cluster models

Abstract: I will discuss exact solvability results (in a sense) for scaling limits of interface crossings in critical random-cluster models in the plane with various boundary conditions. The results are rigorous for the FK-Ising model, Bernoulli percolation, and the spin-Ising model in appropriate setups.
The scaling limit formulas describe structures in the corresponding boundary conformal field theory.

(Based on joint works with Yu Feng, Mingchang Liu, and Hao Wu - all at Tsinghua University, China).

10:40 – 11:10 coffee break

11:10 – 11:55 Denis Bernard (ENS Paris)

Title: The Quantum Symmetric Simple Exclusion Processes

Abstract: An alternative title could have been “How to characterise fluctuations in diffusive out-of-equilibrium many-body quantum systems?” In general, the difficulty to characterise non- equilibrium systems lies in the fact that there is no analog of the Boltzmann distribution to describe thermodynamic variables and their fluctuations. Over the last 20 years, however, it was observed that there is a class of classical non-equilibrium systems with diffusive transport in which the statistics of particle density and particle current show universal properties that do not depend on the microscopic details of the model. The general framework to characterise these systems from a macroscopic point of view is now called the “Macroscopic Fluctuation Theory”. A natural question is whether this framework can be extended to quantum mechanics to describe the statistics of purely quantum mechanical effects such as interference or entanglement in diffusive out-of-equilibrium systems. With this aim in mind, I will introduce the Quantum Symmetric Simple Exclusion Process (Q-SSEP), a microscopic model system, from which we hope to gain inside in possible universal features of fluctuations of those quantum mechanical effects. I will in particular present the recent observation that free cumulants, a tool from free probability theory, seems to play a role in the mathematical structure of this model.

12:00 – 14:00 lunch break

14:00 – 14:45 Antti Knowles (University of Geneva)

Title: Spectral phases of Erdös-Rényi graphs

Abstract: Disordered quantum systems exhibit a variety of spectral phases, characterized by the extent of spatial localization of the eigenvectors. Through their adjacency matrices, random graphs provide a natural class of models for such systems, where the disorder arises from the random geometry of the graph. The simplest random graph is the Erdös-Rényi graph $G(N,p)$, whose adjacency matrix is the archetypal sparse random matrix. The parameter $d = pN$ represents the expected degree of a vertex. A dramatic change in behaviour is known to occur at the scale $d \sim \log N$, which is the threshold where the degrees of the vertices cease to concentrate. Below this scale the graph becomes inhomogeneous and develops structures such as hubs and leaves which accompany the appearance of a localized phase.

I report on recent progress in establishing the phase diagram for $G(N,p)$ at and below the critical scale $d = \log N$. We show that the spectrum splits into a fully delocalized region in the middle of the

spectrum and a semilocalized phase near the spectral edges. The transition between the phases is sharp in the sense of a discontinuity in the localization exponent of eigenvectors. Furthermore, we show that the semilocalized phase consists of a fully localized region and in addition, for some values of d, a complementary region that we conjecture to be nonergodic delocalized. Joint work with Johannes Alt and Raphael Ducatez.

14:50 – 15:35 Valentina Ros (Université Paris-Saclay)

Title: Counting equilibria in high-dimensional random systems: from Gaussian landscapes to simple models of ecosystems

Abstract: High-dimensional random systems often exhibit complex phases in which multiple equilibria of the associated dynamical equations are present. Counting and classifying these equilibria is crucial to understand the systems dynamical evolution. In the field of glasses, equilibria are stationary points of the associated very non-convex (free)-energy landscapes, and several techniques have been developed over the years to count and classify the local minima (metastable states) of these landscapes. These techniques however often involve Boltzmann measures tilted in such a way to pick up the contribution of metastable states: therefore, they explicitly rely on the existence of an underlying landscape. In this talk, I would like to discuss how to compute and classify equilibria in a simple model with non-conservative forces and thus with no associated potential landscape. In particular, I will discuss how to combine the Kac-Rice formalism, random matrix theory and replica theory to compute the typical number of equilibria of generalized Lotka-Volterra equations with random asymmetric interactions.

15:35 – 16:00 coffee break

16:00 – 16:45 Semyon Klevtsov (Université de Strasbourg)

Title: Laughlin states vs. Coulomb gas on Riemann surfaces

Abstract: Fractional quantum Hall effect for simple fractions is famously described by the Laughlin states. The latter are equivalent to the Coulomb gas in the cases of planar and spherical geometries, but differ in interesting ways when the $FQHE$ state is considered on a torus or on a higher-genus Riemann surface. In particular the Laughlin states are degenerate and can be coupled to Aharonov-Bohm fluxes, passing through the holes of the surface. We compute the Chern classes of bundles of Laughlin states over the space of Aharonov-Bohm fluxes, the degeneracy of the Laughlin states with any number of quasi-holes, dimensions of the corresponding Hilbert spaces in higher genus, in particular proving the Wen-Niu conjecture and the Wen-Zee formula. We then argue that the $FQHE$ states are in a "topological phase", when the corresponding bundles are projectively flat. Based on work with Dimitri Zvonkine.

16:50 – 17:35 Paul Wiegmann (University of Chicago)

Title: Dyson gas on a simple curve

The talk is based on the recent paper with Anton Zabrodin where we discussed an ensemble of particles with logarithmic repulsive interaction (the Coulomb gas) on a closed plane contour, a geometric deformation of the Dyson-Selberg integrals. This type of objects arise in the problem of Dyson diffusion, a stochastic process (not necessarily deterministic) with 1/r repulsive force between Brownian movers and many other applications such as SLE.

In the limit of the large number of particles the equilibrium density converges to the harmonic measure of the curve (the Fekete points) and the partition function converges to the spectral determinant of the Neumann jump operator of the contour, or equivalently to the Fredholm determinant of the Neumann–Poincare operator. These results suggest that the deformed Dyson-Selberg integrals utilize the finite dimensional approximation of the complex geometry.

19:00 – conference dinner

Friday, September 30th

9:30 – 10:15 Yan Fyodorov (King's College London)

Title: Extreme complex eigenvalues of a rank-one subunitary deformation of CUE and of its non-Hermitian analogue

Abstract: We study complex eigenvalues most remote from the unit circle in the ensemble of subunitary random matrices of the form
Â=Û diag (1,…,1,√(1-T)), where U∈CUE and T∈(0,1], as well as those most remote from the real line in its non-Hermitian analogue
A=H+iγ diag (1,0,…,0), where γ>0 and H∈GUE.
Both ensembles emerge naturally as random matrix models of wave scattering in quantum chaotic systems, and we discuss similarities and differences between their eigenvalues.

In the subunitary case when the parameter T approaches unity with the rate N^(-1) the eigenvalue with the smallest modulus exhibits a non-trivial extreme value distibution, intermediate between Freche and Gumbel, which we describe explicily. In the non-Hermitian case a nontrivial restructuring of extreme eigenvalues happens when γ approaches unity with the rate N^(-1/3), and we describe this phenomenon at the level of Large Deviations for the mean density and discuss open problems. The presentation is based on joint works with Boris Khoruzhenko and Mihail Poplavskyi

10:20 – 11:05 Roland Bauerschmidt (U Cambridge)

Title: Percolation transition for random forests

Abstract: Given a finite graph, the arboreal gas is the measure on forests (subgraphs without cycles) in which each edge is weighted by a parameter $\beta > 0$. Equivalently this model is bond percolation conditioned to be a forest, the independent sets of the graphic matroid, or the $q \to 0$ limit of the random cluster representation of the q-state Potts model. Our results rely on the fact that this model is also the graphical representation of the non-linear sigma model with target space the fermionic hyperbolic plane $H^{0|2}$. The main question we are interested in is whether the arboreal gas percolates, i.e., whether for a given β the forest has a connected component that includes a positive fraction of the total edges of the graph. We show that in two dimensions a Mermin-Wagner theorem associated with a continuous symmetry of the non-linear sigma model implies that the arboreal gas does not percolate for any $\beta > 0$. On the other hand, in three and higher dimensions, we show that percolation occurs for large β by proving that the symmetry of the non-linear sigma model is spontaneous broken. We also show that the broken symmetry is accompanied by free field like fluctuations (Goldstone mode). This result is achieved by a renormalisation group analysis combined with Ward identities from the internal symmetry of the sigma model. This talk is based on joint works with N. Crawford, T. Helmuth, and A. Swan, and with N. Crawford and T. Helmuth.

11:10 – 11:40 Coffee Break

11:40 – 12:25 Marek Biskup (UCLA)

Title: Limit law for the frequent points of random walks on regular trees

Abstract: A frequent point of a random walk of finite time-length is a point where the walk stays the longest. Considerable effort spanning over six decades went into the understanding of the law of the position of frequent points, and the time spent there, for the simple random walk on hypercubic lattices. Here the case of spatial dimension two turns out to be the most interesting thanks to the approximate scale-invariant nature of the random walk and the logarithmic correlations of the associated local time. This log-correlated structure arises, for similar reasons, also for the random walk on regular trees of finite depth which is technically more accessible due to a Markovian structure of the local time. I will show that, in this case, the time spent at the most frequent leaf of the tree converges, after centering and scaling, to a randomly shifted Gumbel law. Based on joint work with O. Louidor.

12:25 – 14:00 lunch break