Universität zu Köln

Mathematisch-Naturwissenschaftliche Fakultät

Mini-course — **UoC Forum**
"Classical and quantum dynamics of interacting particle systems"

Friday, June 15, 2018
Lecture Hall (Hörsaal) III, Department of Physics
University of Cologne
Zülpicher Straße 77
50937 Köln

Program

9:00 — 10:30 Wolfgang König (TU Berlin, Weierstraß-Institut Berlin)

Large deviations — theory and applications (part 1 of 2)

The theory of large deviations provides probabilistic and analytical tools for describing the exponential decay rate of very small probabilities in terms of variational formulas. The simplest example is the event $A_n = \{S_n \geq tn\}$ with $t \in (0,\infty)$ of the deviation of a centred random walk $(S_n)_{n\in\mathbb{N}}$ from its mean, zero. Cramér's theorem states that the probability of A_n decays like $rm\, e^{-n\, I(t)}$ with some rate function $I(t)$. Even more, $I(t)$ is given by an explicit variational formula, and the minimizers give additional information about the most likely way to realise A_n. This principle is greatly extended to a variety of complex situations like empirical measures of Markov chains, stationary processes and empirical particle fields. We will concisely survey them.

In the development of this theory, driving forces came from various applications in physics, like random path models with interactions having long or short memories, and from static interacting particle systems like the interacting Bose gas. We will explain how to apply the theory to such examples and how much it can contribute to a rigorous understanding of the underlying physics.

The audience should be familiar with the most fundamental concepts of measure theory based probability and with the concept of weak convergence of probability measures.

10:30 — 11:00 Coffee Break

11:00 ─ 12:30 Gunter Schütz (Forschungszentrum Jülich)

Markov Processes and quantum spin chains (part 1 of 2)

The time evolution of the probability distribution of a Markov process is given by a generator in a fashion that is formally very similar to the time evolution of the (complex) probability amplitude of a quantum system in imaginary time. This correspondence allows for the import of technical tools and notions from quantum mechanics to the study of stochastic processes (and vice versa) which has proved to be useful particularly in the context of stochastic interacting particles and quantum spin systems. This mini-course begins with an introduction into the formal aspects of this relationship. In the second part we discuss as applications: (a) Symmetry and duality, (b) Large deviations and Non-hermitian quantum mechanics, and, briefly, (c) Large-scale behaviour.

12:30 ─ 14:00 Lunch Break

14:00 ─ 15:30 Wolfgang König (TU Berlin, Weierstraß-Institut Berlin)

Large deviations ─ theory and applications (part 2 of 2)

15:30 ─ 16:00 Coffee Break

16:00 ─ 17:30 Gunter Schütz (Forschungszentrum Jülich)

Markov Processes and quantum spin chains (part 2 of 2)