Title:

On the Witten Rigidity Theorem for Toeplitz operators

Abstract:

Let M be a closed smooth connected manifold and P be an elliptic differential operator on M. We assume that a compact Lie group G acts on M nontrivially and that P is G-equivariant, by which we mean it commutes with the G action. Then the kernel and cokernel of P are finite dimensional representations of G. The equivariant index of P is the virtual character of G defined by

$$\mathrm{Ind}(h, P) = \mathrm{Tr}\left[h\big|_{\ker P}\right] - \mathrm{Tr}\left[h\big|_{\mathrm{coker}\, P}\right], \text{ for } h \in G. \tag{0.1}$$

P is said to be *rigid* for this G action if $\mathrm{Ind}(h, P)$ does not depend on $h \in G$. It is well known that classical operators: the signature operator for oriented manifolds, the Dolbeault operator for almost complex manifolds and the Dirac operator for spin manifolds are rigid. A highly nontrivial consequence in elliptic genera is that the twisted operator

$$D \otimes T_{\mathbb{C}}M$$

which is known as the *Rarita-Schwinger operator* is rigid.

However, since Dirac operators on odd dimensional manifolds are self ajoint and therefore have index zero, the rigidity for twisted Dirac operators makes sense only for even dimensional manifolds. The appropriate index to consider for an odd dimensional manifold is that of a Toeplitz operator which is defined with the help of an element g in the odd K-group $K^1(M)$.

In this talk, we study the rigidity of Toeplitz operators associated to several Witten type operators. An interesting consequence of our main result is that the *Toeplitz-Rarita-Schwinger operator*

$$\mathcal{T} \otimes T_{\mathbb{C}}M \otimes (\mathbb{C}^N, g)$$

is rigid when g satisfies certain mild topological conditions.

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