**Problems for Hausel's Lecture, Thursday**

1. Identify the Jacobi triple product identity on slide 18 as the Kac denominator formula for the root system $\hat{A}_1$.

2. Prove the statements on slide 19.

3. What is the number of solutions of $x_1y_1 + x_2y_2 + \cdots + x_ny_n = 1$ in $\mathbb{F}_q$? Let $\mathcal{U}$ denote the variety of solutions of this equation. What is the E-polynomial of the variety $\mathcal{U}(\mathbb{C})$? Let $\mathbb{C}^\times$ act on $\mathcal{U}(\mathbb{C})$ by $\lambda : (x_i, y_i) \mapsto (\lambda x_i, \lambda_i^{-1} y_i)$. Show that the action is free and assuming that $\mathcal{U}(\mathbb{C}) \to \mathcal{U}(\mathbb{C})//\mathbb{C}^\times$ is locally trivial in the Zariski topology, calculate the $E$-polynomial of $\mathcal{U}(\mathbb{C})//\mathbb{C}^\times$.
   ( $\mathcal{U}(\mathbb{C})//\mathbb{C}^\times \rightsquigarrow$ Calabi's hyperkähler metric on $T^*\mathbb{P}^{n-1}$)

4. $E(\mathrm{GL}_n(\mathbb{C}); q) =$? Does it have a pure or mixed weight filtration? Can you find the Betti numbers $P_c(\mathrm{GL}_n(\mathbb{C}); t)$ of $\mathrm{GL}_n(\mathbb{C})$? Try to prove that it has the cohomology of the product of certain odd dimensional spheres by looking at a natural tower of fibrations with these spheres in the fibers.

5. Observe that a variety $X$ (defined over $\mathbb{Z}$) which has polynomial count over finite fields and pure weight filtration has $E(X; q) \in \mathbb{N}[q]$.