On Lie algebras associated with representation directed algebras

by

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Let B be a finite dimensional $\mathbb{C}$-algebra and let $\mathrm{ind}(B)$ be a set of representatives of all isomorphism classes of indecomposable B-modules.

With any representation-finite algebra B, Ch. Riedtmann associated the $\mathbb{Z}$-Lie algebra $L(B)$, which is the free $\mathbb{Z}$-module with basis $\{v_X \ ; \ X \in \mathrm{ind}(B)\}$. If B is representation-directed and X, Y are non-isomorphic indecomposable B-modules such that $\mathrm{Ext}^1_B(X, Y) = 0$, the Lie bracket in $L(B)$ is defined by

$$[v_X, v_Y] = \begin{cases} \chi(V(X,Y;Z)) \cdot v_Z & \text{if there is an indecomposable } B-\text{module } Z \\ & \text{and a short exact sequence} \\ & 0 \to X \to Z \to Y \to 0, \\ & \\ 0 & \text{otherwise.} \end{cases},$$

where $\chi(V(X,Y;Z))$ denotes the Euler-Poincaré characteristic of the locally closed subset

$$V(X,Y;Z) = \{\, U \subseteq Z \ ; \ \ U \cong X, \ Z/U \cong Y\}$$

of a product of Grassmann varieties. On the other hand, C. M. Ringel, using Hall polynomials, associated with a representation-directed algebra B the $\mathbb{Z}$-Lie algebra $\mathcal{K}(B)$, which is the free $\mathbb{Z}$-module with basis $\{u_X \ ; \ X \in \mathrm{ind}(B)\}$. If B is representation-directed and X, Y are non-isomorphic indecomposable B-modules such that $\mathrm{Ext}^1_B(X, Y) = 0$, the Lie bracket in $\mathcal{K}(B)$ is defined by

$$[u_Y, u_X] = \begin{cases} \varphi^Z_{YX}(1) \cdot u_Z & \text{if there is an indecomposable } B-\text{module } Z \\ & \text{and a short exact sequence} \\ & 0 \to X \to Z \to Y \to 0 \\ & \\ 0 & \text{otherwise,} \end{cases},$$

where φ^Z_{YX} are Hall polynomials.

We sketch the proof of the following theorem.

Theorem. *Let B be a representation-directed $\mathbb{C}$-algebra. The Lie algebras $L(B)$ and $\mathcal{K}(B)$ are isomorphic.*