**Some results about cohomology of integral Specht modules**

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In 2003, Szczepański formulated the conjecture: For any finite group $G$ there exists a Bieberbach group with holonomy group $G$ and $\mathbb{Q}$-multiplicity free holonomy representation.

In the special case of the symmetric groups, this conjecture can be reformulated in the following way: For any $n \in \mathbb{Z}_{>0}$ there exist distinct partitions $\lambda_1, \ldots, \lambda_m \vdash n$, such that $\bigoplus_{j=1}^{m} S_{\mathbb{Z}}^{\lambda_j}$ is a faithful $\mathbb{Z}S_n$-lattice, and the cohomology group $H^2(S_n, \bigoplus_{j=1}^{m} S_{\mathbb{Z}}^{\lambda_j})$ contains a special element, that is an element $\alpha$ with $\mathrm{res}_U^{S_n}(\alpha) \neq 0$ for any subgroup $U \leq S_n$. ($S_{\mathbb{Z}}^{\lambda_j}$ means the integral Specht module associated to the partition $\lambda_j \vdash n$.)

We discuss some partial results which lead into the direction of a proof of this special case.