Auslander-Reiten theory for modules of finite complexity over selfinjective algebras

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I will report on joint work with Ed Green. Let R be an artin algebra, and M an indecomposable nonprojective R-module. If

$$\cdots \to P^2 \xrightarrow{\delta_2} P^1 \xrightarrow{\delta_1} P^0 \xrightarrow{\delta_0} M \to 0$$

is a minimal projective resolution of M, then the *i-th Betti number* of M, $\beta_i(M)$, is defined to be the number of indecomposable summands of P^i. We say that the *complexity* of a finitely generated R-module M is at most n, and we write

$$\operatorname{cx} M \leq n$$

if $\beta_i(M) \leq ci^{n-1}$, for some $c \in \mathbb{Q}$ and $i \gg 0$ and that the complexity of M is n, $\operatorname{cx} M = n$, if $\operatorname{cx} M \leq n$ but $\operatorname{cx} M \not\leq n-1$. We also say that the complexity of M is infinite, if no n exists such that $\operatorname{cx} M \leq n$. For example, $\operatorname{cx} M = 0$ is equivalent to the projective dimension of M being finite, and $\operatorname{cx} M = 1$ is equivalent to M having infinite projective dimension and the existence of some $b \in \mathbb{Q}$ such that $\beta_n(M) \leq b$, for all $n \geq 0$. It is well-known that if R is a selfinjective artin algebra, then the complexity is constant on each stable component of its Auslander-Reiten quiver. The purpose of this talk is to look at modules of finite complexity over a selfinjective artin algebra using the shape of the Auslander-Reiten sequences ending at them.