Conference: Perspectives in Representation Theory
27 September - 03 October 2009, Cologne

ABSTRACTS

—— MAIN SPEAKERS

Hall algebras of cyclic quivers and affine quantum Schur algebras

BANGMING DENG
Beijing Normal University, China

In this talk we first define surjective algebra homomorphisms from double Hall algebras of cyclic quivers to affine quantum Schur algebras and then discuss the structure of affine quantum Schur algebras. This is based on a joint work with J. Du and Q. Fu.

Moment graphs in representation theory and topology

PETER FIEBIG
University of Freiburg, Germany

We give an introduction to the theory of moment graphs and its applications towards multiplicity conjectures in representation theory and modular smoothness of Schubert varieties.

Onesketeton galleries and Hall polynomials

STÉPHANE GAUSSENT
University of Nancy, France

I will report on a joint work with Peter Littelmann. To any finite Weyl group, one can associate the algebra of symmetric polynomials. It has two natural bases: the Schur polynomials and the monomial polynomials. The Hall-Littlewood ones form another basis that interpolates between those two. Using a geometrical interpretation of them, we obtain a combinatorial formula for the coefficients appearing in the expansion of Hall-Littlewood polynomials in terms of monomial ones. This formula is in the same spirit as the one that Schwer proved. Except that we use oneskeleton galleries instead of galleries of alcoves, therefore it should be thought of as a compression of Schwer's one.

Some problems in the representation theory of Hecke algebras

Meinolf Geck
University of Aberdeen, UK

Hecke algebras arise naturally in the representation theory of finite groups of Lie type, as endomorphism algebras of certain induced representations. We will discuss some important open questions in this area, most notably James' Conjecture and Lusztig's Conjectures on Hecke algebras with unequal parameters.

Brauer algebras and their Schur algebras

Anne Henke
University of Oxford, UK

Schur-Weyl duality relates the representation theory of two algebras. In 1937, Brauer asked the following question: which algebra has to replace the group algebra of the symmetric groups in the set-up of Schur-Weyl duality if one replaces the general linear group by its orthogonal or symplectic subgroup. As an answer he defined an algebra which is a special case of what nowadays is called Brauer algebra.

In the situation of the symmetric groups Σ_r and general linear groups GL_n, Schur algebras are defined as the ring of those endomorphisms of the r-fold tensor space of an n-dimensional vector space that commute with the symmetric group action. Alternatively, a Morita equivalent version of the Schur algebra can be defined as the endomorphism ring of permutation modules for symmetric groups.

In the situation of the Brauer algebras and orthogonal/symplectic groups, the two definitions indicated above lead to different Schur algebras. The talks will discuss Schur algebras corresponding to Brauer algebras, in particular the Schur algebras defined via permutation modules.

Lie theory for exotic finite dihedral groups

Michail Kapovich
University of California, Davis, USA

I will describe our work with Arkady Berenstein on developing a "Lie Theory" where the role of Weyl groups is played by arbitrary dihedral groups. In particular, I will talk about geometry of associated spherical and Euclidean buildings, decomposition of tensor products and a candidate construction for the actual infinite-dimensional Lie algebra.

Numerical equalities and categorical equivalences in block theory

Radha Kessar
University of Aberdeen, UK

I will discuss some approaches to and results on the structure of blocks of modular group algebras.

Cluster algebras and representation theory

Bernard Leclerc
University of Caen, France

Cluster algebras have been introduced by Fomin and Zelevinsky in 2001, motivated by various combinatorial problems arising notably in Lie theory. After a quick introduction to cluster algebras, I will review a series of joint works with C. Geiss and J. Schröer about cluster algebra structures on coordinate rings of unipotent groups and flag varieties. In the last lecture I will discuss a recent joint paper with D. Hernandez which shows that certain tensor categories of representations of quantum affine algebras also have interesting cluster structures.

Endo-trivial modules for finite groups

Jacques Thévenaz
EPFL Lausanne, Switzerland

1. Endo-trivial modules for finite groups. Endo-trivial modules for a finite group G are representations of G in characteristic p which play an important role in modular representation theory and block theory. In the case of a p-group, they have been classified in 2004. This talk will give an introduction to the subject.

2. Endo-trivial modules: present and future. After the classification of all endo-trivial modules for p-groups, several results have been obtained recently for other groups. However, a classification in general seems to be hard. This talk will give an overview on some of the questions involved in the subject.

Highest weight categories which are Calabi-Yau-0

Will Turner
University of Aberdeen, UK

Category theory is a notoriously pointless subject. If we wish to make it more pointed, we must place strong restrictions on the theorised categories. Two strong categorical restrictions are the highest weight restriction and the Calabi-Yau-0 restriction. The highest weight restriction shows up in the representation theory of algebraic groups, whilst the Calabi- Yau-0 restriction shows up in the representation theory of finite groups. I will discuss recent work on categories in which both of these restrictions are assumed to hold. There are connections to representation theory, the theory of tilings, and algebraic geometry.

ABSTRACTS
—— SHORT TALKS

Irreducible characters of groups associated with finite involutive algebras

Carlos André
University of Lisbon, Portugal

An algebra group is a group of the form G = 1+J where J = J(A) is the Jacobson radical of a finite-dimensional associative algebra A (with identity). A theorem of Z. Halasi asserts that, in the case where A is defined over a finite field F, every irreducible complex representation of G is induced by a linear representation of a subgroup of the form H = 1+J(B) for some subalgebra B of A. If we assume that F has odd characteristic p and (A,s) is an algebra with involution, then s naturally defines a group automorphism of G = 1+J, and we may consider the fixed point subgroup G(s). In this situation, we show that every irreducible complex representation of G(s) is induced by a linear representation of a subgroup of the form H(s) where H = 1+J(B) for some s-invariant subalgebra B of A. As a particular case, the result holds for the Sylow p-subgroups of the classical groups of Lie type.

On graded Lie algebras

Hannes Bierwirth
University of Málaga, Spain

I study the Lie structure of graded associative algebras. We derive, as consequences, examples of algebras of quotients of graded Lie algebras and show that the Lie algebra of graded derivations of a graded associative algebra is graded strongly nondegenerate when the center does not contain nozero associative ideals.

Multi-component KP construction and the WDVV equation

Evgeny Feigin
Lebedev Physics Institute, Moscow, Russia

The celebrated construction of the semi-infinite wedge space due to M.Jimbo and T.Miwa allows to study certain system of differential equations (called the KP hierarchy) in terms of the representation theory of the Lie algebra of infinite matrices. It turned out that the natural multi-component generalization of the semi-infinite wedge space can be used in order to construct solutions of the WDVV (associativity) equation. In the talk we will briefly recall main points of the notions and constructions above. We will also make a link between the representation theory of certain loop groups (acting on the wedge space)and the formalism of Frobenius structures due to A.Givental.

The depth of subalgebras and subgroups

BURKHARD KÜLSHAMMER
University of Jena, Germany

The depth is a numerical invariant which can be attached to a Frobenius extension of rings. In the talk, we will explore this notion in the context of finite-dimensional algebras, in particular group algebras. This will be a report on joint work with R. Boltje, S. Burciu, S. Danz and L. Kadison.

A minimal generating system for semi-invariants of quivers for dimension $(2, \ldots, 2)$

ARTEM LOPATIN
University of Bielefeld, Germany

We work over an infinite field K of arbitrary characteristic. Given a quiver Q, we denote by $I(Q, \underline{n})$ ($SI(Q, \underline{n})$, respectively) the algebra of invariants (semi- invariants, respectively) of representations of Q for dimension vector $\underline{n}$. The generators for $SI(Q, \underline{n})$ are known. In this talk we will explicitly describe a minimal (by inclusion) generation system for $SI(Q, \underline{n})$ for $\underline{n} = (2, \ldots, 2)$.

Note that the known generating system for $I(Q, \underline{n})$ is "simpler" than that for $SI(Q, \underline{n})$. Moreover, the ideal of relations between generators for invariants is known in contrast to semi-invariants. Nevertheless, a minimal generating system for $I(Q, (2, \ldots, 2))$ is not known and it is unclear how it can be explicitly described.

Finite arithmetic groups and Galois operation

DMITRY MALININ
University of Vlora, Albania

We consider a Galois extension E/F of characteristic 0 and realization fields of finite abelian subgroups $G \subset GL_n(E)$ of a given exponent t. We assume that G is stable under the natural operation of the Galois group of E/F. It is proven that under some reasonable restrictions for n any E can be a realization field of G, while if all coefficients of matrices in G are algebraic integers there are only finitely many fields E of realization having a given degree d for prescribed integers n and t or prescribed n and d.

Below O_E is the maximal order of E and $F(G)$ is an extension of F generated via adjoining to F all matrix coefficients of all matrices $g \in G$, Γ is the Galois group of E over F.

We prove the existence of abelian Γ-stable subgroups G such that $F(G) = E$ provided some reasonable restrictions on the fixed normal extension E/F and integers n, t, d hold and study the interplay between the existence of Γ-stable groups G over algebraic number fields and over their rings of integers.

Let K be a totally real algebraic number field with the maximal order O_K, G an algebraic subgroup of the general linear group $GL_n(\mathbf{C})$ defined over the field of rationals $\mathbf{Q}$. Since G can be embedding to $GL_n(\mathbf{C})$ the intersection $G(O_K)$ of $GL_n(O_K)$ and $G(K)$, the subgroup of K-rational points of G, can be considered as the group of O_K-points of an affine group scheme over $\mathbf{Z}$, the ring of rational integers. Assume G to be definite in the following sense: the real Lie group $G(\mathbf{R})$ is compact. The problem which is our starting point is the question: Does the condition $G(O_K) = G(\mathbf{Z})$ always hold true?

This problem is easily reduced to the following conjecture from the representation theory: Let $K/\mathbf{Q}$ be a finite Galois extension of the rationals and $G \subset GL_n(O_K)$ be a finite subgroup stable under the natural operation of the Galois group $\Gamma := Gal(K/\mathbf{Q})$. Then there is the following

Conjecture 1. *If K is totally real, then $G \subset GL_n(\mathbf{Z})$.*

There are several reformulations and generalizations of the conjecture. Consider an arbitrary not necessarily totally real finite Galois extension K of the rationals $\mathbf{Q}$ and a free $\mathbf{Z}$-module M of rank n with basis $m_1, \ldots, m_n$. The group $GL_n(O_K)$ acts in a natural way on $O_K \otimes M \cong \bigoplus_{i=1}^n O_K m_i$. The finite group $G \subset GL_n(O_K)$ is said to be of A-type, if there exists a decomposition $M = \bigoplus_{i=1}^k M_i$ such that for every $g \in G$ there exists a permutation $\Pi(g)$ of $\{1, 2, \ldots, k\}$ and roots of unity $\epsilon_i(g)$ such that $\epsilon_i(g) g M_i = M_{\Pi(g)i}$ for $1 \leq i \leq k$. The following conjecture generalizes (and would imply) conjecture 1:

Conjecture 2. *Any finite subgroup of $GL_n(O_K)$ stable under the Galois group $\Gamma = Gal(K/\mathbf{Q})$ is of A-type.*

For totally real fields K conjecture 2 reduces to conjecture 1.

Both conjectures are true in the case of Galois field extension $K/\mathbf{Q}$ with odd discriminant. Also some partial answers are given in the case of field extensions $K/\mathbf{Q}$ which are unramified outside 2.

The following result was obtained in [1] (see also [2], [4] for the case of totally real fields).

The case $F = \mathbf{Q}$, the field of rationals, is specially interesting. The following theorem was proven in [1] using the classification of finite flat group schemes over $\mathbf{Z}$ annihilated by a prime p obtained by V. A. Abrashkin and J.- M. Fontaine:

Theorem 1. *Let $K/\mathbf{Q}$ be a normal extension with Galois group Γ, and let $G \subset GL_n(O_K)$ be a finite Γ-stable subgroup. Then $G \subset GL_n(O_{K_{ab}})$ where K_{ab} is the maximal abelian over $\mathbf{Q}$ subfield of K.*

Finiteness Theorem. *1) For a given number field F and integers n and t, there are only a finite number of normal extensions E/F such that $E = F(G)$ and G is a finite abelian Γ-stable subgroup of $GL_n(O_E)$ of exponent t.*

2) For a given number field F and integers n and d, there is only a finite number of fields E such that $d = [E\colon F]$ and $E = F(G)$ for some finite Γ-stable subgroup G of $GL_n(O_E)$.

Theorem 2. *Let F be a field of characteristic 0, let $d > 1, t > 1$ and $n \geq \phi_E(t)d$ (here $\phi_E(t)d = [E(\zeta_t) : E]$ is the generalized Euler function, , ζ_t is a primitive t-root of 1) be given integers, and let E be a given normal extension of F having the Galois group Γ and degree d. Then there is an abelian Γ- stable subgroup $G \subset GL_n(E)$ of the exponent t such that $E = F(G)$.*

In fact, G can be generated by matrices g^γ, $\gamma \in \Gamma$ for some $g \in GL_n(E)$.

Theorem 3. *Let E/F be a given normal extension of algebraic number fields with the Galois group Γ, $[E : F] = d$, and let $G \subset GL_n(E)$ be a finite abelian Γ-stable subgroup of exponent t such that $E = F(G)$ and n is the minimum possible. Then $n = d\phi_E(t)$ and G is irreducible under conjugation in $GL_n(F)$. Moreover, if G has the minimum possible order, then G is a group of type $(t, t, ..., t)$ and order t^m for some positive integer $m \leq d$.*

In the case of quadratic extensions we can give an obvious example.

Example. *Let $d = 2$, $t = 2$. Set $E = \mathbf{Q}(\sqrt{a})$ and $g = \begin{vmatrix} 0 & 1 \\ a^{-1} & 0 \end{vmatrix} \sqrt{a}$ for any $a \in F$ which is not a square in F. Then Γ is a group of order 2 and $G = \{I_2, -I_2, g, -g\}$ is a Γ-stable abelian group of exponent 2.*

In the case of unramified extensions the following theorem for integral representations in a similar situation is proven in [3]:

Theorem 4. *Let $d > 1, t > 1$ be given rational integers, and let E/F be an unramified extension of degree d.*

1)If $n \geq \phi_E(t)d$, there is a finite abelian Γ- stable subgroup $G \subset GL_n(O'_E)$ of exponent t such that $E = F(G)$ where O'_E is the intersection of valuation rings of all localization rings of O_E with respect to primes ramified in E/F.

2)If $n \geq \phi_E(t)dh$ and h is the exponent of the class group of F, there is a finite abelian Γ-stable subgroup $G \subset GL_n(O_E)$ of exponent t such that $E = F(G)$.

3)If $n \geq \phi_E(t)d$ and h is relatively prime to n, then any G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.

4)If d is odd, then any G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.

In all cases above G can be constructed as a group generated by matrices $g^\gamma, \gamma \in \Gamma$ for some $g \in GL_n(E)$.

References

[1] H.-J. Bartels, D. A. Malinin, "Finite Galois stable subgroups of GL_n." In: Noncommutative Algebra and Geometry, Edited by C. de Concini, F. van Oystaeyen, N. Vavilov and A. Yakovlev, Lecture Notes In Pure And Applied Mathematics, vol. 243 (2006), p. 1–22.

[2] D. A. Malinin, "Galois stability for integral representations of finite groups". Algebra i analiz, vol 12 (2000), p.106–145.

[3] D.A.Malinin, " On the existence of finite Galois stable groups over integers in unramified extensions of number fields ". Publ. Mathem. Debrecen, v.60/1-2 (2002), p. 179–191.

[4] D.A.Malinin, "Integral representations of finite groups with Galois action ", Dokl. Russ. Akad. Nauk, v.349 (1996), p.303–305.

Verifying the abelian defect group conjecture for sporadic simple groups

Jürgen Müller
RWTH Aachen, Germany

This talk will emphasize the interplay between theoretical and computational techniques. This is a joint work with S. Koshitani and F. Noeske.

Matching Simple Modules of Condensed Algebras

Felix Noeske
RWTH Aachen, Germany

Let A be a finite dimensional algebra over a finite field F. Condensing an A-module V with two different idempotents e and e' leads to the problem that to compare the composition series of Ve and Ve', we need to match the composition factors of both modules. In other words, given a composition factor S of Ve, we have to find a composition factor S' of Ve' such that there exists a composition factor T of V with Te = S and Te' = S', or prove that no such S' exists.

In this talk, we present a computationally tractable solution to this problem, and illustrate how its application was a crucial step in the recent proof of the 3-modular character table of the sporadic simple Harada Norton group.

Modular Characters for Brauer algebras

Armin Shalile
University of Oxford, UK

We define modular characters for Brauer algebras which share many of the features of Brauer characters defined for groups. Since notions such as conjugacy classes and orders of elements are not a priori meaningful for Brauer algebras, we show which structure replaces the conjugacy classes and determine eigenvalues associated to these.