Workshop on cellular and diagram algebras in mathematics and physics – Titles and abstracts of talks

Georgia Benkart: *Perfect Crystals - Imperfect Questions and Conjectures*
Perfect crystals have remarkable properties. This talk will discuss some recent work with Frenkel, Kang, and Lee giving a uniform construction of level one crystals for all affine Lie algebras and what they might be trying to tell us.

Karin Erdmann: *Stratifying Cellular Algebras*
Let R be a cellular algebra, with cell modules W(\lambda). Very often, the cell modules form a stratifying system as it was introducedin joint work with Corina Saenz a while ago. In particular this is so for group algebras of symmetric groups, Hecke algebras of type A and Brauer algebras, exluding a few small characteristics or small roots of unity as parameters. An important conclusion is that in this case, for modules with cell filtrations, filtration multiplicities are well defined. Amongst others, for such algebras there are always analogues of Young modules. This lecture will introduce stratifying systems and discuss important properties.

Richard M Green: *Constructing cell data for diagram algebras*
Tabular algebras are a class of associative ${\Bbb Z}[v, v^{-1}]$-algebras equipped with distinguished bases (tabular bases) and satisfying certain axioms. The two main applications of tabular algebras are (a) as a framework for studying Kazhdan--Lusztig type bases, and (b) as templates for studying cellular algebras. Many of the examples of cellular structures for algebras studied in the literature are in practice constructed by starting from a tabular basis and applying a general trick. The main purpose of these talks is to explain exactly what this trick is and why it works, and to discuss various examples.

Sofia Lambropoulou: *Yokonuma-Hecke algebras and invariants of framed links*
We first introduce the Yokonuma-Hecke algebras and we then show how to construct two infinite series of Markov traces related to framed braids. These normalized give invariants of framed links. (Joint work with Jesus Juyumaya)

Bernard Leclerc: *Preprojective algebras and Kac-Moody algebras*
To a finite unoriented graph without loop we can associate
(1) a Lie algebra : the Kac-Moody algebra
(2) an associative algebra : the preprojective algebra,
introduced by Gelfand and Ponomarev in 1980.
I shall explain a joint work with C. Geiss and J. Schr\"oer in which we construct the irreducible integrable modules and Verma modules over the Kac-Moody algebra in terms of injective modules over the preprojective algebra. This is inspired by (and generalizes) some results of Lusztig. I shall then sketch an application (in the A,D,E case) : how to understand certain "clusters" of Berenstein, Fomin and Zelevinsky in terms of very special modules over the preprojective algebra.

Gus Lehrer: Diagrams and cellularity in associative algebras.
I shall give some instances where diagrammatic realisations of algebras permit an analysis of their representation theory through cellular methods. Particular applications are to the representation theory of affine and other Hecke algebras.
First lecture: Diagrammatic versions of various Temperley-Lieb algebras, both finite and infinite dimensional; analysis of their representation theory. Algebraic definitions of some Hecke and TL-algebras.
Second lecture: Connections between diagrammatic and algebraic versions of the algebras; representation theoretic consequences for affine Hecke algebras and quivers. Affine Hecke algebras as modules over their centre, and cellularity.

Paul Martin: *On diagram algebras and their role in statistical mechanics (a selective review)*
This is a kind of review talk (an 'underview', perhaps). We discuss relationships between diagram algebras and various problems in statistical mechanics, with the aim that representation theory might be informed by placing it in the Physical context.
More diagram algebras and statistical mechanics
We elaborate on some of the paradigms introduced in the first talk, and tie in some more recent

developments.

Arun Ram: *Murphy elements for diagram algebras*
I will present a recent theorem of M.Mazzocco giving new Murphy elements for Temperley-Lieb algebras, explain a general method for deriving Murphy elements using as an example the generalizations of the partition algebras corresponding to complex reflection groups, and discuss the application of Murphy elements to the determination of the quiver of the algebra.

Changchang Xi: *Homological techniques for cellular algebras*
In this talk I will report some developments on cellular algebras, which show that homological techniques and methods are useful for understanding cellular algebras and diagram algebras as well as their combinatorial properties. For example, homological global dimension of a cellular algebra is finite characterizes that thedecomposition matrix is a square matrix. As application of the methods, one can determine for which parameters the Birman-Wenzl algebras could have maximal numbers of non-isomorphic simple modules.
The materials for this talk will cover some of works of myself and my students as well as works jointly with Steffen Koenig, Hebing Rui, and others.

Marcos Alvarez: *Towards an algebra of 2-manifolds surgery*

Matthew Bloss: *G-Colored Partition Algebras as Centralizer Algebras of Wreath Products*
Let G be a group. We define an associative algebra $P_k(n;G)$ that is a partition algebra whose diagrams have oriented edges labeled by elements of G. For G finite, we show that $P_k(n;G)$ is the centralizer algebra of an action of the wreath product $G \wr S_n$ on tensor powers of its permutation module.

Anton Cox: *Representation theory of towers of recollement*
We give an axiomatic framework for studying the representation theory of towers of algebras, and illustrate its applicability to a wide class of diagram algebras. This is joint work with Paul Martin, Alison Parker and Changchang Xi.

Stephen Doty: *Rational Schur algebras*
(Joint work with R. Dipper)
Rational Schur algebras are a generalization of the ordinary Schur algebras in type A. They control (through finite dimensional algebra techniques) all the rational representations of general linear groups (over an infinite field) in the same way that the ordinary Schur algebras control the polynomial representations. They provide new examples of generalized Schur algebras (in the sense of Donkin) and new examples of
quasihereditary algebras. Their centralizer algebras are certain subalgebras of the Brauer algebra, spanned by a certain distinguished collection of Brauer diagrams. Needless to say, this entire picture quantizes nicely.

Alice Fialowski: *The moduli space of 4-dimensional Lie algebras*
We will show that the moduli space of complex 4-dimensional Lie algebras is essentially an orbifold, given by the natural action of the symmetric group S_3 on the complex projective space P^2(C). In addition, there are two exceptional complex lines, one of which has an action of the symmetric group S_2, and 6 exceptional points. The moduli space is glued together by the miniversal deformations, which determine the elements that one may deform to locally, so deformation theory determines the geometry of the space.

John Hall: *Deformations of the Full Transformation Semigroup*
The full transformation semigroup T_n is the semigroup of all maps from a set of size n to itself. Its semigroup algebra $\C T_n$ is a subalgebra of the partition algebra $P_n(Q)$, albeit one in which the parameter Q does not appear. The representation theory of T_n is closely tied to that of the symmetric group S_n, but the usual questions are considerably more difficult to answer because $\C T_n$ is not semisimple. In this talk we exhibit a deformation of $\C T_n$ and discuss a generically semisimple

algebra that results.

Tom Halverson: *Robinson-Schensted Insertion for Diagram Algebras*
We will look at the Robinson-Schensted-Knuth algorithm for the symmetric group and how it extends in two different ways to diagram algebras, including the Brauer algebra, the partition algebra, and rook monoid algebras. The first RSK algorithm describes the decomposition of the regular representation into irreducibles. The second describes the decomposition of the diagram algebra as a tensor power centralizer algebra via Schur-Weyl duality.

Anne Henke: *Diagram algebras and stratifications*
Let A be a diagram algebra, such as a Brauer, partition or Temperley-Lieb algebra. We study homological properties of A, using filtrations by idempotent ideals AeA. We show Ext-comparisons and Ext-vanishing properties similar to those of quasi-hereditary algebras, as well as an analogue of a theorem by Hemmer and Nakano.
This is joint work with Robert Hartmann, Steffen Koenig and Rowena Paget.

Masashi Kosuda: *Standard basis for the party algebra P_{n,r}(Q)*
The party algebra P_{n,r}(k) is the centralizer algebra of the unitary reflection group G(r, 1, k) in the tensor space. This algebra is a subalgebra of the partition algebra which is (in our notation) the centralizer of G(1,1,k) in the tensor space. Recently Naruse and I found a good standard basis for the party algebra P_{n,r}(Q), which will give a cellular representation and conjugacy class of P_{n,r}(Q). In the talk, I will talk about the topic above.

Sinead Lyle: *Row removal theorems for homomorphisms of Weyl modules*
(Joint work with Andrew Mathas)
There exists a (generalised) analogue of James's row and column removal theorems concerning homomorphisms between Specht modules and between Weyl modules. The proof of this result given by myself and Andrew Mathas relies on the cellular basis of the q-Schur algebra.

Hiroshi Naruse: *Characters of the party algebras*
We generalize the results of T.Halverson about the characters of partition algebra P_n(d) to the case of party algebra P_{n,r}(d). For example "Character table", Frobenius formula, and Murnaghan-Nakayama rule are described in a similar way. We also point out inter-relations between character tables in various r's, and decomposition rule of restriction to S_n.

Alexander Nichols: *One-boundary Temperley-Lieb algebras in the XXZ and loop models*
This talk is based on cond-mat/0411512 in collaboration with V. Rittenberg and J. de Gier
We give an exact spectral equivalence between the quantum group invariant XXZ chain with arbitrary left boundary term and the same XXZ chain with purely diagonal boundary terms.
This equivalence, and a further one with a link pattern Hamiltonian, can be understood as arising from different representations of the one-boundary Temperley-Lieb algebra. For a system of size L these representations are all of dimension 2^L and, for generic points of the algebra, equivalent. However at exceptional points they can possess different indecomposable structures.
We study a centralizer of the one-boundary Temperley-Lieb algebra in the 'non-diagonal' spin-1/2 representation and find its eigenvalues and eigenvectors. In the exceptional cases the centralizer becomes indecomposable. We show how to get a truncated space of 'good' states. The indecomposable part of the centralizer leads to degeneracies in the three mentioned Hamiltonians.

Jan Okninski: *Algebraic structure of the Chinese algebras*
For a positive integer n and a field K we consider the associative algebra $R =K\langle a_{1},\ldots , a_{n}\rangle$ defined by the presentation:
$$a_{j}a_{i}a_{k}=a_{j}a_{k}a_{i}= a_{k}a_{j}a_{i} \, \mbox{ for }
\, i<k<j$$ and
$$a_{i}a_{j}a_{i}=a_{j}a_{i}a_{i}, \quad a_{j}a_{i}a_{j}=a_{j}a_{j}a_{i}
\, \mbox{ for } \, i<j.$$ It is the semigroup algebra of the Chinese monoid, whose combinatorial properties were studied in \cite{chinese}. This monoid is related to the so called plactic monoid, first considered in \cite{las-schut}, see also \cite{las-lec}. Both constructions are strongly related to Young tableaux, and therefore to representation theory and algebraic combinatorics, and the latter construction has already

become a classical tool in these theories, \cite{fulton}.
The structure of the Chinese algebra R is studied. As a partial answer to certain more general questions, the minimal prime ideals are described and the classical Krull dimension is computed in case R is of rank 3. It follows that every minimal prime ideal is determined by a homogeneous congruence on M. Moreover, the Jacobson radical of R is nilpotent. Problems of these type are also motivated by a general program of studying finitely presented algebras defined by homogeneous semigroup relations.
This is a joint work with Joanna Jaszu\'nska.
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Uri Onn: *On cellular Hecke algebras associated with Grassmannians*
Let A be a finite O-module, where O is a discrete valuation ring. Let b be an isomorphism type of a submodule of A. Then G=Aut(A) acts on the Grassmannian Gr(b,A), and gives rise to a linear representation of G on V=Fun(Gr(b,A)). We analyze such representations using the structure of the lattice of submodules of A. Under restricted assumptions on A and b, the Hecke algebra associated to this representation, i.e. End_G(V), is shown to be cellular. Its idempotents can be computed explicitly. (Joint work with Uri Bader.)

Rowena Paget: *Defining Young modules for Brauer algebras*
Brauer algebras were introduced by R. Brauer in 1937 as type B,C,D analogues of group algebras of symmetric groups (in characteristic zero). More recently Graham and Lehrer, and later Koenig and Xi, showed that, defined over any field, Brauer algebras (like symmetric group algebras) are cellular algebras. The cell modules for the Brauer algebra play the role of the Specht modules for the symmetric group. We look for Brauer algebra modules which are analogues of the permutation modules, and of the Young modules. We also obtain an analogue of the Hemmer-Nakano theorem for symmetric groups. This is joint work with Robert Hartmann.

Catharina Stroppel: *Cell modules and tensor products of finite dimensional modules of quantum sl_2.*
We give a categorical interpretation of cell modules of (Iwahahori) Hecke algebras giving rise to a categorification of irreducible modules for the symmetric group in characcteristic zero. The basis of such modules will be identified with isomorphism classes of tilting modules in parabolic versions of category O.
On the other side we give a categorical interpretation of tensor products of finite dimensional quantum sl_2 modules using Harish Chandra modules. The dual canonical basis will be described via simple modules, whereas the canonical basis has an interpretation in terms of projective objects.

Masha Vlasenko: *The affine Temperley-Lieb algebra*
We give a realization of the affine Temperley-Lieb algebra by matrices over the ring of Laurent polynomials and consider corresponding traces.

Bruce Westbury: *Diagrams and crystal graphs.*
In this talk I will build on work of Greg Kuperberg who gave a diagrammatic description of the centraliser algebras of the seven dimensional representation of the exceptional simple Lie algebra G_2. A consequence of this is that, for n>0, we have two finite sets whose size is the dimension of the space of invariant tensors in the n-th tensor power of this representation. One set is a set of walks on the set of dominant weights, the other set is the set of non-positive planar trivalent graphs embedded in the disc with n boundary points. In this talk I will describe inverse bijections between these two sets.
The main idea is to use the PRV theorem to interpret the vertices of the crystal graph of the representation as diagrams. If time permits I will show that this idea can be appplied to representations of simple Lie algebras of arbitrary rank.