Universität zu Köln
Mathematisches Institut
Prof. Dr. F. Vallentin
Dr. A. Gundert
A. Heimendahl

Methods and problems in discrete mathematics

Wintersemester 2019/20

— Exercise Sheet 2 —

Exercise 2.1 Make the following statement precise: The linear programming bound for potential energy generalizes the linear programming bound for spherical codes[1].

$$\begin{aligned} \inf \quad & \lambda \\ & \beta_0, \beta_1, \ldots, \beta_d \geq 0 \\ & \sum_{k=0}^{d} \beta_k = \lambda - 1 \\ & \sum_{k=0}^{d} \beta_k P_n^k(t) \leq -1 \text{ for } t \in [-1, \cos\gamma]. \end{aligned}$$

Hint: If you're stuck, a look into the Cohn-Kumar paper might help.

Exercise 2.2 Consider the sequence (p_n) of orthogonal polynomials satisfying the orthogonality relation

$$\int_{-1}^{1} p_m(x) p_n(x)\, dx = 0 \text{ for } m \neq n,$$

normalized by $p_n(1) = 1$. Determine p_0, p_1, p_2, p_3, p_4 and verify the 3-term recurrence relation for p_2, p_3 and p_4.

Exercise 2.3 Determine $H(f, g)$ for

$$f(t) = 1/(2-2t)^3 \quad \text{and} \quad g(t) = (t+1)^2(t+1/2)^2 t^2 (t-1/2)^2.$$

Exercise 2.4 Present the proof of Proposition 2.2 (see page 110 in the Cohn-Kumar paper): Let f be absolutely monotonic in (a, b), let $g(t) = (t-t_1)^{k_1} \cdots (t-t_m)^{k_m}$. Then, $Q(f, g)(t)$ is again absolutely monotonic in (a, b).

"Hand-in": Until Thursday October 24, 10 am, using the form on the course homepage.

[1] A *spherical code* with minimal angle γ is a point configuration $\mathcal{C} \subseteq S^{n-1}$ so that $x \cdot y \in [-1, \cos\gamma]$ whenever $x \neq y$.