Universität zu Köln
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Methods and problems in discrete mathematics

Wintersemester 2019/20

— Exercise Sheet 4 —

Exercise 4.1 Let $K_i : X \times X \to \mathbb{C}$ for $i = 1, 2$ be two G-invariant, positive-definite kernels. Show that $K(x, y) = K_1(x, y) \cdot K_2(x, y)$ is again a G-invariant, positive-definite kernel.

Exercise 4.2 Let $\hat{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$. For any matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}_2(\mathbb{R})$ let

$$Ax = \frac{ax + b}{cx + d},$$

where $x \in \hat{\mathbb{R}}$, with the convention that $\frac{a\infty+b}{c\infty+d} = \frac{a}{c}$ and $\frac{u}{0} = \infty$ for $u \neq 0$. Show that this defines an action of $\mathrm{GL}_2(\mathbb{R})$ on $\hat{\mathbb{R}}$.

Exercise 4.3 Let G be a group acting on a set X. With $x \in X$ we associate the *orbit* of x

$$\mathrm{Orb}(x) = \{gx \mid g \in G\}$$

and the *stabilizer subgroup* of G with respect to x

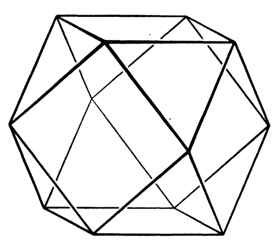
$$\mathrm{Stab}(x) = \{g \in G \mid gx = x\}.$$

(a) Let G be a finite group acting on a finite set X and let $x \in X$. Show:

$$|\mathrm{Orb}(x)| = |G|/|\mathrm{Stab}(x)|.$$

(b) Determine the order of the symmetry group of the cuboctahedron P,

$$G = \{A \in \mathrm{O}(3) : AP = P\}.$$



The cuboctahedron (from Hilbert, Cohn-Vossen, Anschauliche Geometrie, Springer, 1932)

Exercise 4.4 Let G be a finite abelian group and let $(V, \langle\cdot,\cdot\rangle)$ be an irreducible unitary representation of G. Show that $\dim V = 1$.

“Hand-in”: Until Thursday **November 14**, 10 am, using the form on the course homepage.