Universität zu Köln
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Methods and problems in discrete mathematics

Wintersemester 2019/20

— Exercise Sheet 6 —

Exercise 6.1 Show that $\mathrm{Harm}_k \subseteq \mathbb{C}[x_1, x_2]_k$ is spanned by the polynomials

$$(x_1 + ix_2)^k \text{ and } (x_1 - ix_2)^k.$$

Exercise 6.2 Show that $\mathcal{C} \subseteq S^{n-1}$ is a spherical t-design if and only if for all $f \in \mathrm{Harm}_k$ with $k \in \{1, \ldots, t\}$ we have $\sum_{x \in \mathcal{C}} f(x) = 0$.

Exercise 6.3 Prove the following linear programming bound for spherical t-designs: Let $\mathcal{C} \subseteq S^{n-1}$ be a spherical t-design. Suppose there is a polynomial h such that

(i) $h(u) \geq 0$ for all $u \in [-1, 1]$,

(ii) $h(u) = P_0^n(u) + \sum_{k=1}^{d} \alpha_k P_k^n(u)$ with $\alpha_1, \ldots, \alpha_d \in \mathbb{R}$ and with $\alpha_{t+1}, \ldots, \alpha_d \leq 0$,

holds. Then, $|\mathcal{C}|$ is at least $h(1)$.

Exercise 6.4 Compute the linear programming bound numerically for spherical t-designs with parameters $n = 2, \ldots, 12$ and $t = 4, 5, 6$.

"Hand-in": Until Thursday **November 28**, 10 am, using the form on the course homepage.