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Methods and problems in discrete mathematics

Wintersemester 2019/20

— Exercise Sheet 7 —

Exercise 7.1 For an orthonormal basis $e_1, \dots, e_{h_k}$ of H_k we define the zonal spherical function

$$z_k : S^{n-1} \times S^{n-1} \to \mathbb{C} \quad \text{by} \quad z_k(x, y) = \sum_{i=1}^{h_k} e_i(x)\overline{e_i(y)}.$$

Show:

(a) z_k does not dependent on the choice of the orthonormal basis of H_k.

(b) For all $A \in \mathrm{O}(n)$ and for all $x, y \in S^{n-1}$ we have $z_k(Ax, Ay) = z_k(x, y)$.

Exercise 7.2 Compute Δf for

$$f(x_1, \dots, x_n) = \sum_{i=0}^{k/2} c_i x_1^{k-2i}(x_2^2 + \cdots + x_n^2)^i.$$

Exercise 7.3 Let $G = (V, E)$ be a graph.

(a) Assume that G is k-regular. Show that the multiplicity of λ_1, the largest eigenvalue of the adjacency matrix of G, equals the number of connected components of G.

(b) Suppose there are r vertices which all have the same neighbours. Show that 0 is an eigenvalue of the adjacency matrix of G which has multiplicity at least $r - 1$.

Exercise 7.4 Let $G = (V, E)$ be a graph.

(a) Show that there is a non-negative eigenvector for $\lambda_1(G)$, the largest eigenvalue of the adjacency matrix of G.

(b) Let G' be a subgraph of G. Show that $\lambda_1(G') \leq \lambda_1(G)$.

"Hand-in": Until Thursday **December 5**, 10 am, using the form on the course homepage.