Universität zu Köln
Mathematisches Institut
Prof. Dr. F. Vallentin
Dr. A. Gundert
A. Heimendahl

Methods and problems in discrete mathematics

Wintersemester 2019/20

— Exercise Sheet 10 —

Exercise 10.1 Consider the set $X = \{1, 2, \dots, n\}$ and the set of pairs $P = \{(i, j) : i, j \in X, i < j\}$. Every $p \in \mathbb{R}^P$ defines a symmetric function $d_p \in \mathbb{R}^{X \times X}$ by

$$d_p(i,j) = \begin{cases} p(i,j) & \text{if } i < j, \\ 0 & \text{if } i = j, \\ p(j,i) & \text{if } i > j. \end{cases}$$

Let

$$M_n = \{p \in \mathbb{R}^P : d_p \text{ is a pseudometric on } X\},$$

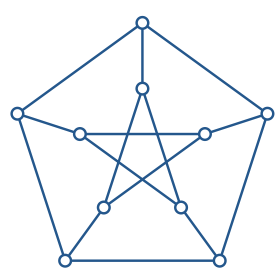
where a non-negative and symmetric function $d \in \mathbb{R}^{X \times X}$ is a *pseudometric* if $d(x, x) = 0$ for all $x \in X$ and d satisfies the triangle inequality.
Show that M_n is a polyhedral cone. Sketch the cone M_3 and determine its extreme rays.

Exercise 10.2 Let (X, d), with $X = \{x_1, \dots, x_n\}$, be a finite metric space. Show that

$$c_2(X,d) \geq \max_{Y \in \mathcal{S}^n_+, Ye=0} \sqrt{\frac{\sum_{ij:Y_{ij}>0} Y_{ij} d(x_i,x_j)^2}{-\sum_{ij:Y_{ij}<0} Y_{ij} d(x_i,x_j)^2}},$$

where $e = (1, \dots, 1)^{\mathsf{T}}$.

Exercise 10.3 What is the minimal distortion embedding of the Petersen graph?



Exercise 10.4 Consider the symmetric group S_n. The graph G_n has vertex set $V = S_n$, and two vertices $\sigma, \pi \in S_n$ are adjacent if and only if σ is the composition of π and a transposition that swaps consecutive elements. What is the minimal distortion embedding of G_4?

"Hand-in": Until Thursday **January 9**, 10 am, using the form on the course homepage.