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Methods and problems in discrete mathematics

Wintersemester 2019/20

— Exercise Sheet 11 —

Exercise 11.1

(a) Consider the statement from Exercise 10.2. Show that in the case of equality we have $Y_{ij} > 0$ only for the most contracted pairs (x_i, x_j), and $Y_{ij} < 0$ only for the most expanded pairs (x_i, x_j).

(b) Let G be a graph. Show that in any Euclidean embedding of G the most expanded pair (x, y) is attained at adjacent vertices x and y.

Exercise 11.2 Compute $c_2(C_n)$, where C_n is the n-cycle graph.

Exercise 11.3 Compute the minimal Euclidean distortion of the vertex-edge-graph of the cross polytope in dimension n.
The cross polytope is defined as $\operatorname{conv}\{\pm e_1, \ldots, \pm e_n\}$, where $e_i \in \mathbb{R}^n$ is the i-th standard basis vector.

Exercise 11.4 Present a proof of Dirac's theorem on Hamiltonian cycles: Let G be a simple graph with $n \geq 3$ vertices where every vertex has degree at least $n/2$. Then G contains a Hamiltonian cycle.

"Hand-in": Until Thursday **January 16**, 10 am, using the form on the course homepage.