Universität zu Köln
Mathematisches Institut
Prof. Dr. F. Vallentin
G. Fischer
Dr. M.C. Zimmermann

Convex Optimization

Winter Term 2020/21

— Exercise Sheet 2 (November 10, 2020) —

Exercise 2.1. Prove the following lemma: If K is a proper convex cone, then its dual cone K^* is proper.

Exercise 2.2. Prove the following lemma: Let K be a closed, full-dimensional convex cone. Then x lies in the interior of K if and only if $x^{\mathsf{T}}y > 0$ for all $y \in K^* \setminus \{0\}$.

Exercise 2.3. Show that the set of non-negative polynomials of degree at most $2d$

$$\left\{(a_0, a_1, \ldots, a_{2d}) \in \mathbb{R}^{2d+1} : a_0 + a_1 x + \cdots + a_{2d}x^{2d} \geq 0 \text{ for all } x \in \mathbb{R}\right\}$$

is a proper convex cone for any $d \geq 0$.

Exercise 2.4. Prove the following theorem of alternatives: Let $K \subseteq \mathbb{R}^n$ be a proper convex cone, and let $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$. Then exactly one of the following two alternatives holds:

(a) Either there exists $x \in K \setminus \{0\}$ such that $Ax = 0$ and $c^{\mathsf{T}}x \geq 0$,

(b) or there exists $y \in \mathbb{R}^m$ such that $A^{\mathsf{T}}y - c \in \text{int } K^*$.