Universität zu Köln
Mathematisches Institut
Prof. Dr. F. Vallentin
G. Fischer
Dr. M.C. Zimmermann

Convex Optimization

Winter Term 2020/21

— Exercise Sheet 4 (November 24, 2020) —

Exercise 4.1. Let $C \in \mathcal{S}^n$ and $k \in \mathbb{N}$ be given. Consider the following maximization problem

$$\max_{X \in \mathcal{S}^n} \{\langle C, X\rangle : \mathrm{Tr}(X) = k, I_n \succeq X \succeq 0\}.$$

(a) Write the given problem as a semidefinite program.

(b) Determine the corresponding dual program.

Exercise 4.2. What is the **exact** value of the minimal maximal eigenvalue of the following matrix $X \in \mathcal{S}^5$

$$\begin{pmatrix} 1 & X_{12} & 1 & X_{14} & X_{15} \\ X_{12} & 1 & 1 & X_{24} & X_{25} \\ 1 & 1 & 1 & X_{34} & 1 \\ X_{14} & X_{24} & X_{34} & 1 & 1 \\ X_{15} & X_{25} & 1 & 1 & 1 \end{pmatrix} ?$$

Hint: You can use a numerical SDP solver to "guess" the solution.

Exercise 4.3. Let $X, Y \in \mathcal{S}^n$ be symmetric matrices. Determine the minimum

$$\min\{\langle X, AYA^{\mathsf{T}}\rangle : A \in \mathsf{O}(n)\}.$$

Exercise 4.4. For which $k \in \mathbb{N}$ is the function

$$\Phi_k : \mathcal{S}^n \to \mathbb{R},\ X \mapsto \mathrm{Tr}(X^k)$$

a convex spectral function?