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Methoden und Probleme der diskreten Mathematik

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— Aufgabenblatt 4 —

Aufgabe 4.1 Sei Δ ein n-Simplex mit den Ecken $V = \{v_1, \ldots, v_{n+1}\}$.
Zeigen Sie: F ist eine Seite von Δ genau dann wenn es ein $W \subseteq V$ gibt mit $F = \operatorname{conv}(W)$.

Aufgabe 4.2
Sei $h = \{(\alpha, \beta) \in \mathbb{Z}^2 \mid \alpha - \beta = 0\}$ und definiere die Funktionen

$$f(x,y) = \sum_{\substack{(\alpha,\beta)\in h\\ \alpha,\beta\geq 0}} x^\alpha y^\beta \quad \text{und} \quad \overline{f}(x,y) = \sum_{\substack{(\alpha,\beta)\in h\\ \alpha,\beta> 0}} x^\alpha y^\beta.$$

Zeigen Sie dass $\overline{f}(x,y) = -f(\frac{1}{x}, \frac{1}{y})$ gilt, wenn man f und $\overline{f}$ als rationale Funktionen betrachtet.

Aufgabe 4.3 Zeigen Sie

$$\frac{1}{(1-z)^{n+1}} = \sum_{k\geq 0} \binom{n+k}{n} z^k.$$

Aufgabe 4.4 Für ein Polytop $P \subseteq \mathbb{R}^2$ sei A die Fläche, I die Anzahl der inneren Gitterpunkte und B die Anzahl der Gitterpunkte auf dem Rand.

a) Zeigen Sie die Gleichung $A = I + \frac{1}{2}B - 1$ für ganzzahlige achsenparallele Rechtecke und ganzzahlige Dreiecke mit zwei achsenparallelen Seiten.

b) Zeigen Sie dass $L_P(t) = At^2 + \frac{1}{2}Bt + 1$ gilt für ganzzahlige Polytope $P \subseteq \mathbb{R}^2$.

Abgabe: Bearbeitete Aufgaben bis spätestens Mittwoch, den 05. November 2014 um 23 Uhr 59, in das Onlineformular auf der Vorlesungshomepage eintragen.

— Zitate —

Aus dem Aufsatz "Discrete and Continuous: Two sides of the same?" von László Lovász:

There are different levels of interaction between discrete and continuous mathematics, and I treat them (I believe) in the order of increasing significance.

1. We often use the finite to approximate the infinite. To discretize a complicated continuous structure has always been a basic method – from the definition of the Riemann integral through triangulating a manifold in (say) homology theory to numerically solving a partial differential equation on a grid.

   It is a slightly more subtle thought that the infinite is often (or perhaps always?) an approximation of the large finite. Continuous structures are often cleaner, more symmetric, and richer than their discrete counterparts (for example, a planar grid has a much smaller degree of symmetry than the whole euclidean plane). It is a natural and powerful method to study discrete structures by "embedding" them in the continuous world.

2. Sometimes, the key step in the proof of a purely "discrete" result is the application of a purely "continuous" theorem, or vice versa. [. . .]

3. In some areas of discrete mathematics, key progress has been achieved through the use of more and more sophisticated methods from analysis. [. . .] (I could not find any area of "continuous" mathematics where progress would be achieved at a similar scale through the introduction of discrete methods. Perhaps algebraic topology comes closest.)

4. Connections between discrete and continuous may be the subject of mathematical study on their own right. *Numerical analysis* may be thought of this way, but *discrepancy theory* is the best example. [. . .]

5. The most significant level of interaction is when one and the same phenomenon appears in both the continuous and discrete setting. In such cases, intuition and insight gained from considering one of these may be extremely useful in the other. A well-known example is the connection between sequences and analytic functions, provided by the power series expansion. In this case, there is a "dictionary" between combinatorial aspects (recurrences, asymptotics) of the sequence and analytic properties (differential equations, singularities) of its generating function.