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Methoden und Probleme der diskreten Mathematik

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— Aufgabenblatt 14 —

Aufgabe 14.1 Erklären Sie (in eigenen Worten) das Theorem von Roth über die Existenz von 3-APs und seinen Beweis, der Fourier-Analysis verwendet, mit minimal 300 und maximal 500 Wörtern.

Aufgabe 14.2 Erklären Sie (in eigenen Worten) das Theorem von Roth über die Existenz von 3-APs und seinen Beweis, der das Regularitätslemma verwendet, mit minimal 300 und maximal 500 Wörtern.

Abgabe: keine.

— Zitate —

Aus: Terence Tao — *What is good mathematics?*

We all agree that mathematicians should strive to produce good mathematics. But how does one define “good mathematics”, and should one even dare to try at all? Let us first consider the former question. Almost immediately one realises that there are many different types of mathematics which could be designated “good”. For instance, “good mathematics” could refer (in no particular order) to

- (i) Good mathematical *problem-solving* (e.g. a major breakthrough on an important mathematical problem);
- (ii) Good mathematical *technique* (e.g. a masterful use of existing methods, or the development of new tools);
- (iii) Good mathematical *theory* (e.g. a conceptual framework or choice of notation which systematically unifies and generalises an existing body of results);
- (iv) Good mathematical *insight* (e.g. a major conceptual simplification, or the realisation of a unifying principle, heuristic, analogy, or theme);
- (v) Good mathematical *discovery* (e.g. the revelation of an unexpected and intriguing new mathematical phenomenon, connection, or counterexample);
- (vi) Good mathematical *application* (e.g. to important problems in physics, engineering, computer science, statistics, etc., or from one field of mathematics to another);
- (vii) Good mathematical *exposition* (e.g. a detailed and informative survey on a timely mathematical topic, or a clear and well-motivated argument);
- (viii) Good mathematical *pedagogy* (e.g. a lecture or writing style which enables others to learn and do mathematics more effectively, or contributions to mathematical education);
- (ix) Good mathematical *vision* (e.g. a long-range and fruitful program or set of conjectures);
- (x) Good mathematical *taste* (e.g. a research goal which is inherently interesting and impacts important topics, themes, or questions);
- (xi) Good mathematical *public relations* (e.g. an effective showcasing of a mathematical achievement to non-mathematicians, or from one field of mathematics to another);
- (xii) Good *meta-mathematics* (e.g. advances in the foundations, philosophy, history, scholarship, or practice of mathematics);
- (xiii) *Rigorous* mathematics (with all details correctly and carefully given in full);
- (xiv) *Beautiful* mathematics (e.g. the amazing identities of Ramanujan; results which are easy (and pretty) to state but not to prove);
- (xv) *Elegant* mathematics (e.g. Paul Erdős’ concept of “proofs from the Book”; achieving a difficult result with a minimum of effort);
- (xvi) *Creative* mathematics (e.g. a radically new and original technique, viewpoint, or species of result);
- (xvii) *Useful* mathematics (e.g. a lemma or method which will be used repeatedly in future work on the subject);
- (xviii) *Strong* mathematics (e.g. a sharp result that matches the known counterexamples, or a result which deduces an unexpectedly strong conclusion from a seemingly weak hypothesis);
- (xix) *Deep* mathematics (e.g. a result which is manifestly non-trivial, for instance by capturing a subtle phenomenon beyond the reach of more elementary tools);
- (xx) *Intuitive* mathematics (e.g. an argument which is natural and easily visualisable);
- (xxi) *Definitive* mathematics (e.g. a classification of all objects of a certain type; the final word on a mathematical topic);
- (xxii) etc., etc.¹

¹The above list is not meant to be exhaustive. In particular, it focuses primarily on the type of mathematics found in mathematical research papers, as opposed to classrooms, textbooks, or papers in disciplines close to mathematics, such as the natural sciences.