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BMS Summer School: Convex Geometry — Discrete and Computational

Packings, Coverings, and Embeddings

— Problem Sheet 2: Embeddings —

Problem 2.1 = Problem 1.4*

Problem 2.2 = Problem 1.5

Problem 2.3 Let $G = (V, E)$ be a d-regular graph with n vertices and let

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$$

be the eigenvalues of the adjacency matrix of G. Then:

a) $\lambda_i \in [d, -d]$ for all $i = 1, \ldots, n$.

b) G is connected if and only if $\lambda_1 > \lambda_2$.

c) G is bipartite if and only if $\lambda_1 = -\lambda_n$.

d) $\lambda_2^2 \geq d\frac{n-d}{n-1}$.

Problem 2.4 Let $G = (V, E)$ be a strongly regular graph with parameters (v, k, λ, μ). What is $c_2(G)$?

Problem 2.5* Let $G = (V, E)$ be a strongly regular graph with parameters $(v, k, 0, 1)$. Then there are four possibilities for v and k:

$$(v, k) = (5, 2),\ (10, 3),\ (50, 7),\ (3250, 57).$$

It is currently not known whether a strongly regular graph with parameters $(3250, 57, 0, 1)$ exists.

Problem 2.6** Let $G = (V, E)$ be a graph having girth g. It is conjectured that

$$c_2(G) = \Omega(g)$$

holds. The best lower bound which is currently known is $c_2(G) = \Omega(\sqrt{g})$.

Problem 2.7** Let $G = (V, E)$ be a skeleton of a polytope. Does $c_2(G) = O(\sqrt{\log |V|})$ hold?