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BMS Summer School: Convex Geometry — Discrete and Computational

Packings, Coverings, and Embeddings

— Problem Sheet 4: Packings —

Problem 4.1 What is the value of $\vartheta(\text{Cayley}(\mathbb{Z}_{11}, \{1, -1, 5, -5\}))$?

Problem 4.2 Let χ and ψ be characters of $\mathbb{Z}_n$. Then the following orthogonality relation holds:

$$\chi^*\psi = \sum_{x \in \mathbb{Z}_n} \overline{\chi(x)}\psi(x) = \begin{cases} |\mathbb{Z}_n| & \text{if } \chi = \psi, \\ 0 & \text{otherwise.} \end{cases}$$

Problem 4.3 Given a finite graph $G = (V, E)$ and a weight function $w \colon V \to \mathbb{R}_{\geq 0}$, we define

$$\begin{array}{lll} \vartheta'_w(G) = \min & M & \\ & K(x,x) \leq M & \text{for all } x \in V, \\ & K(x,y) \leq 0 & \text{for all } \{x,y\} \notin E \text{ with } x \neq y, \\ & K \in S^V, & \\ & K - (w^{1/2})(w^{1/2})^{\mathsf{T}} \text{ is positive semidefinite,} & \end{array} \tag{1}$$

where $w^{1/2} \in \mathbb{R}^V$ is such that $w^{1/2}(x) = w(x)^{1/2}$. What is the dual of this?

Problem 4.4 Let $G = (V, E)$ be a *regular* (i.e. e is an eigenvector of the adjacency matrix A) and *edge-transitive* graph (i.e. for each two edges $e = \{x, y\}$ and $e' = \{x', y'\}$ of G there is an automorphism $\sigma \in \text{Aut}(G)$ with $\sigma(e) = \{\sigma(x), \sigma(y)\} = e'$). Then the following equation holds:

$$\vartheta(G) = |V| \frac{-\lambda_{\min}}{\lambda_{\max} - \lambda_{\min}},$$

where $\lambda_{\min}$ is the smallest and $\lambda_{\max}$ the largest eigenvalue of A.