Universität zu Köln
Mathematisches Institut
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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 1 —

Exercise 1.1. Let $f\colon \mathbb{R}^n \to \mathbb{R}$ be a convex function such that there is a $C \in \mathbb{R}$ with $f(x) \leq C$ for all $x \in \mathbb{R}^n$. Prove that f has to be a constant function.

Exercise 1.2.

1. Show that
$$f(x_1, \ldots, x_n) = \begin{cases} -\left(\prod_{i=1}^{n} x_i\right)^{1/n} & \text{if } x_1, \ldots, x_n \geq 0, \\ \infty & \text{otherwise.} \end{cases}$$
is a convex function.

2. Let $f, g\colon \mathbb{R}^n \to \mathbb{R}$ be convex functions. Show that $h(x) = \max\{f(x), g(x)\}$ is convex.

Exercise 1.3. Let $X = (X_{ij})_{ij}$ be a 2×2 positive definite matrix. There is a unique lower triangular matrix $L = (L_{ij})_{ij}$ with $L_{11}, L_{22} > 0$ and $X = LL^{\mathsf{T}}$ (the Cholesky factorization of X). Show that the negative of the matrix entry L_{22} is a convex function in the variables X_{11}, X_{12}, and X_{22}.
First Hint. $-L_{22}(X_{11}, X_{12}, X_{22}) = -\sqrt{X_{22} - \frac{X_{12}^2}{X_{11}}}$.
Second Hint. Why can you "ignore" the square root here?

Exercise 1.4. Consider the minimization problem

$$\begin{aligned} \min\ & \frac{1}{2} x^{\mathsf{T}} P x + q^{\mathsf{T}} x + r \\ & x \in \mathbb{R}^n \end{aligned}$$

where $P \in \mathbb{R}^{n \times n}$ is a symmetric matrix, $q \in \mathbb{R}^n$ is a vector, and $r \in \mathbb{R}$ is a real number.

1. Show that if P is not positive semidefinite (i.e. the objective function is not convex), then the problem is unbounded below.

2. Suppose that P is positive semidefinite (so the objective function is convex), but the optimality condition $Px^* = -q$ does not have a solution. Show that the problem is unbounded below.

Hand-in: Until Tuesday, 27th October, 2pm at the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.