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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 2 —

Exercise 2.1.

1. Show that
$$K = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : x_1 \geq 0, x_2 \geq 0, x_1 x_2 \geq x_3^2 \right\}$$
is a proper convex cone.
2. Determine the dual cone of $K = \text{cone}\{v_1, \dots, v_N\}$ with $v_1, \dots, v_N \in \mathbb{R}^n$.

Exercise 2.2. Let $K \subseteq \mathbb{R}^n$ be a proper convex cone. Show that its dual cone K^* is a proper convex cone, as well.

Exercise 2.3. Consider the convex cone
$$L_p^{n+1} = \{(x, t) \in \mathbb{R}^{n+1} : \|x\|_p \leq t\},$$
where $\|x\|_p = (|x_1|^p + \cdots + |x_n|^p)^{1/p}$ is the p-norm of x. Determine its dual cone $(L_p^{n+1})^*$ verifying the special case in example 2 b).
Hint. Use Hölder's inequality.

Exercise 2.4. Let $K \subseteq \mathbb{R}^n$ be a proper convex cone. Show that x lies in the topological interior of K if and only if $y^{\mathsf{T}} x > 0$ holds for all $y \in K^* \setminus \{0\}$.

Hand-in: Until Tuesday, 3rd November, 2pm at the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.