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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 3 —

Exercise 3.1. Show that the topological interior of the cone of positive semidefinite matrices consists out of the positive definite matrices: $\operatorname{int} \mathcal{S}^n_{\succeq 0} = \mathcal{S}^n_{\succ 0}$.

Exercise 3.2. Let $X \in \{\pm 1\}^{n \times n}$ be a symmetric matrix whose entries are 1 or -1. Show that X is positive semidefinite if and only if $X = xx^{\mathsf{T}}$ for some $x \in \{\pm 1\}^n$.

Exercise 3.3. Given $x_1, \ldots, x_n \in \mathbb{R}$, consider the following matrix

$$X = \begin{pmatrix} 1 & x_1 & \ldots & x_n \\ x_1 & x_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ x_n & 0 & 0 & x_n \end{pmatrix}.$$

That is, $X \in \mathcal{S}^{n+1}$ is the matrix indexed by $\{0, 1, \ldots, n\}$, with entries $X_{00} = 1$, $X_{0i} = X_{i0} = X_{ii} = x_i$ for $i \in \{1, \ldots, n\}$, and all other entries are equal to 0.
Show that X is positive semidefinite if and only if $x_i \geq 0$ for all $i \in \{1, \ldots, n\}$ and $\sum_{i=1}^n x_i \leq 1$.

Exercise 3.4. Consider the matrix

$$X = \begin{pmatrix} a & b & c & d \\ b & b & d & d \\ c & d & c & d \\ d & d & d & d \end{pmatrix} \in \mathcal{S}^4.$$

Show that X is positive semidefinite if and only if the following four linear inequalities are fulfilled

$$\begin{aligned} &a - b - c + d \geq 0 \\ &b - d \geq 0 \\ &c - d \geq 0 \\ &d \geq 0. \end{aligned}$$

Hand-in: Until Tuesday, 10th November, 2pm at the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.